For all real numbers $a$ and $b$, the following intervals are defined. The notation on the LEFT is called “interval notation”. The set on the RIGHT is the DEFINITION of the interval on the left, using the usual set-builder notation. (See Additional Notes on notation below).

\[
\begin{align*}
[a, b] &= \{ x \in \mathbb{R} \mid a \leq x \leq b \} \\
(a, b] &= \{ x \in \mathbb{R} \mid a < x \leq b \} \\
[a, b) &= \{ x \in \mathbb{R} \mid a \leq x < b \} \\
(a, b) &= \{ x \in \mathbb{R} \mid a < x < b \} \\
[a, \infty) &= \{ x \in \mathbb{R} \mid x \geq a \} \\
(a, \infty) &= \{ x \in \mathbb{R} \mid x > a \} \\
(-\infty, b] &= \{ x \in \mathbb{R} \mid x \leq b \} \\
(-\infty, b) &= \{ x \in \mathbb{R} \mid x < b \}
\end{align*}
\]

Each of the numbers $a$ and $b$ is called an endpoint of the respective interval. Intervals which contain all of their endpoints — $[a, b]$, $[a, \infty)$, and $(-\infty, b]$ — are called closed, while those which contain none of their endpoints are open.

**Additional Notes.**

For the first four definitions above, it is often (but not always) assumed that $a \leq b$, sometimes that $a < b$.

In this class this semester, we will always assume, in these cases, that $a$ and $b$ are chosen so that the interval defined is not empty. (On occasion, we may assume more.)

A basic property of an interval, one which could be used to define “interval” in general, is:

A set $J$ is an interval iff

If $a$ and $b$ are elements of $J$, then every real number between $a$ and $b$ is in $J$.

As usual, this last statement could also be expressed as

For all $a$ and $b$ which are elements of $J$, every real number between $a$ and $b$ is in $J$.

Note that this definition, as stated, would include a set containing only one point as an interval, and the empty set as an interval. If you don’t want these sets to be intervals, you have to change the definition as given here by ruling them out.

This definition also implies that the set $\mathbb{R}$ of all real numbers is an interval, often denoted by $(-\infty, \infty)$. 