For emphasis (this is a general guideline; it will not be repeated in every Lesson): **Be efficient in your proofs** (e.g., don't repeat work that you've already done; **use previous results** where appropriate).

1. Prove the following facts (a) to (f) (see definitions above)

   For practice, **you should not use the fact that an integer is odd iff it is not even:**

   a. If \( n \) is an even integer and \( m \) is an integer, then \( nm \) is even.

      **COMMENT.** This will be discussed in class. Note the instruction above (RTI): So do not use the fact that …

   b. If \( n \) is an even integer and \( m \) is an integer, then \( mn \) is even.

      **PROOF.** Suppose \( n \) is an even integer and \( m \) is an integer. Then, by (a), \( nm \) is even.

      By commutativity, \( mn = nm \). So, \( mn \) is even.

      **OR, a very slightly different approach:**

      Suppose \( n \) is an even integer and \( m \) is an integer. Consider \( mn \).

      By commutativity, \( mn = nm \). By (a), \( nm \) is even. So, \( mn \) is even.

   c. If \( m \) is an even integer and \( n \) is an integer, then \( mn \) is even.

      (As noted above, keep the proof as simple as possible.)

      **PROOF.** This is exactly the same statement as (a) above (with the variables \( m \) and \( n \) interchanged). Since (a) is true, this is true.

      [**COMMENT:** Commutativity is irrelevant here. You have already proved, in (a), exactly this statement.]

      *May be continued with (d) (e) (f).*