3. **Comments** on how Problem 3 was graded (the statements to be analyzed are repeated below). For each statement, a correct negation (labeled appropriately) earned 6 points. For a) and b), 4 points each for NOT giving a converse or contrapositive. For c), 4 points each for converse and contrapositive. For each statement, -2 points for not giving the original statement (the instructions for the problem requested that the statement be written on your paper). Write the given statement on your paper before your discussion, so it is clear what you’re talking about.

   a) 6 is even or not prime.
   b) 27 is odd or 17172717 is prime.
   c) If 6 is not even, then 17172717 is prime.

4. For each of the statements in the preceding problem, determine whether the statement is true or false, and explain your conclusion efficiently and concisely using arithmetic and logic. Write each statement on your paper before your discussion of that statement.

   **SOLUTION.** (a) is true, because “6 is even” is true (arithmetic), and an “or” statement is true if at least one part of it is true (logic).
   
   (b) is true, because “7 is odd” is true (arithmetic), so the logic of (a) applies.
   
   (c) is true because “6 is not even” is false (arithmetic); this is the antecedent of the given statement, and “if-then” statement is true if the antecedent is false (logic).

   **COMMENT.** Note that none of the explanations requires analysis of 17172717, and so explanations which are efficient and concise will ignore 17172717.

5. All variables refer to real numbers. For each of the statements below, give the negation (NEN), decide whether the given statement is true, and prove your answer about truth.

   a. $\exists y \in [0, 3)$ such that $\forall x \in [0, 3), y \leq x$.

      **SOLUTION.** Negation: $\forall y \in [0, 3), \exists x \in [0, 3) \text{ such that } y > x$.

      The original statement **(a) is true.** **Proof.** Choose $y = 0$. Consider $x$ in $[0, 3)$. Then (by definition of the interval $[0, 3)$) $x \geq 0$, so $x \geq y$. Thus $y \leq x$.

   b. $\forall x \in [0, 3), \exists y \in [0, 3)$ such that $y > x$.

      **SOLUTION.** Negation: $\exists x \in [0, 3) \text{ such that } \forall y \in [0, 3), y \leq x$.

      The original statement **(b) is true.** **Proof.** Assume $x \in [0, 3)$. Since $x < 3$ (by definition of the interval $[0, 3)$), we can find a number $y$ between $x$ and 3.

      *But it is best to be specific:* E.g., Choose $y = (x+3)/2$.

      Then $0 \leq x < y < 3$ (this can be verified with simple algebra), so $y \in [0, 3)$ and $y > x$.

   c. $\exists y \in [0, 3)$ such that $\forall x \in [0, 3), x \leq y$.

      **SOLUTION.** Negation: $\forall y \in [0, 3), \exists x \in [0, 3) \text{ such that } x > y$.

      Note that, mathematically and logically, this negation (for (c)) is exactly the same as the original statement (b). Since (b) is true, this negation is true, and therefore **(c) is false.**