3. (Included for reference, for use in Problem 4 below.)
   We will say, for this exam, that a number is **flat** iff it is an integer which is a multiple of 5.
   **Prove** the following using the definition of “is a multiple of” (all variables refer to integers):
   
   (a) 0 is flat. (I.e., \(5 \mid 0\).)
   (b) \(n\) is flat iff \(-n\) is flat.
   (c) If \(a\) is flat and \(b\) is flat, then \(a + b\) is flat.

4. Equivalence Relations. (Slightly different notation than used on exam.)
   Define a relation \(\sim\) on \(\mathbb{R}\) by
   \[ x \sim y \text{ iff } x \text{ and } y \text{ are real and } x - y \text{ is an integer which is flat } (i.e., 5 \mid (x - y)). \]
   **Prove** that this is an equivalence relation on \(\mathbb{R}\) (of course, use the definition of the relation to do this).

   **Proof.** Use Problem 3:
   Reflexive: Suppose \(x\) is a real number. Then \(x - x = 0\), which is flat by 3(a).
   Thus \(x \sim x\). So the relation is reflexive.
   
   Symmetric: Suppose \(x \sim y\). Then, by definition of the relation, \(x - y\) is flat.
   Now, \(y - x = -(x - y)\), so by 3(b), \(y - x\) is flat.
   So \(y \sim x\). Thus, the relation is symmetric.
   
   Transitive. Suppose \(x \sim y\) and \(y \sim z\). Then, by definition of the relation, \(x - y\) and \(y - z\) are flat.
   Now, \(x - z = (x - y) + (y - z)\), so by 3(c), \(x - z\) is flat.
   So \(x \sim z\). Thus, the relation is transitive.

5. Equivalence Classes.
   For the relation defined in the preceding problem, think about the equivalence class of \(\pi\)
   (denoted by \([\pi]\)). If nonempty, **give the three smallest positive elements** of this equivalence class.
   (Explain your answer if you say “empty”;
   if nonempty, explain why the elements you give are in \([\pi]\), but you don’t have to explain why positive and smallest.)

   **SOLUTION.** Three smallest positive elements of \([\pi]\) are \(\pi\), \(\pi + 5\), \(\pi + 10\).
   Each of these is in \([\pi]\) because each is equivalent to \(\pi\), because the difference between \(\pi\) and each of
   these three numbers is a multiple of 5.

   **COMMENT.** We have a relation on the real numbers. Since \(\pi\) is a real number, there is an equivalence
   class \([\pi]\) which consists of all real numbers \(x\) with \(x \sim \pi\); three of these are given above.

6. Discussed elsewhere.

7. Division Algorithm.
   Suppose \(n\) is an integer. Consider 5 as a divisor (the number we have denoted by \(d\) when
   discussing the Division Algorithm). **Write down** what the Division Algorithm says about \(n\) with
divisor 5. (“There exist ... .”) Don’t forget “uniqueness”.
   (What is the Division Algorithm for “\(n\) divided by 5” ?)

   **SOLUTION.** There exist integers \(q\) and \(r\) such that \(n = q(5) + r\) and \(0 \leq r < 5\).
   For any given \(d\), there is only one such pair \((q, r)\).
8. (a) List the elements of the set \( \{ r \in \mathbb{Z} : 0 \leq r < 5 \} \) as shown below:

\[
SOLUTION. \quad \{ r \in \mathbb{Z} : 0 \leq r < 5 \} = \{ 0, 1, 2, 3, 4 \}.
\]

(b) Decide whether \([0, 5) = \{ r \in \mathbb{Z} : 0 \leq r < 5 \}\).

\[
\text{If true, just write the equality on your paper and say “true”.}
\]

\[
\text{If false, prove false.}
\]

\[
SOLUTION. \quad \text{False. Consider } x = 4.5. \text{ Then } x \text{ is in the interval } [0, 5), \text{ but } x \text{ is not in } \{ 0, 1, 2, 3, 4 \}
\]

9. Suppose \( m \) and \( b \) are real numbers.

(a) Consider the set \( \{ (x, y) \in \mathbb{R} \times \mathbb{R} : \text{for all real } u \text{ and } v, \quad v = mu + b \} \).

What is this set? Give a simple description and prove or at least explain clearly.

\[
SOLUTION. \quad \text{The statement after the colon (:)} \text{ in the set-builder description of the set is a false statement. Thus, the set is empty.}
\]

(b) Discuss \( \{ (x, y) \in \mathbb{R} \times \mathbb{R} : \text{for all real } x \text{ and } y, \quad y = mx + b \} \).

\[
SOLUTION. \quad \text{First interpretation: Bad notation, since the } x \text{ and } y \text{ after the colon are being used in a different way (dummy variables) than the } x \text{ and } y \text{ before the colon.}
\]

\[
\text{Second interpretation: If we assume that the } x \text{ and } y \text{ after the colon are intended to be different variables than those before the colon, then this is the same as the set in (a).}
\]