7. Let $F$ be a function defined as follows: For each real number $x$, $F(x)$ is the interval $(x, \infty)$.

(a) Let $\mathbb{R}$ be the set of all real numbers. Decide whether the power set of $\mathbb{R}$ (i.e., the set of all subsets of $\mathbb{R}$) is an appropriate codomain for $F$, and explain.

**SOLUTION.** Each output of the function $F$ is an interval, and therefore a subset of $\mathbb{R}$. Thus, the set of all subsets of $\mathbb{R}$ is an appropriate codomain.

(b) Decide if this function, with this codomain, is surjective, and explain your answer clearly.

**SOLUTION.** Generally, there are subsets of $\mathbb{R}$ which are not intervals, and therefore cannot be outputs of this function.

Specifically, choose any ONE of the following subsets of $\mathbb{R}$: $\emptyset$, $\{7\}$, $\mathbb{R}$ (for example). None of these is an output of the function. So the power set of $\mathbb{R}$ is not the range, and so $F$ is not surjective.

(c) Decide if this function is injective, and explain your answer clearly.

**SOLUTION.** Suppose $a$ and $b$ are real numbers and $F(a) = F(b)$.

Then, by definition of $F$, $(a, \infty) = (b, \infty)$. Thus $a = b$. (This can be accepted. But a proof, using trichotomy (implicitly) is not hard.) So $F$ is injective, by definition of injective.