Suppose $n$ and $d$ are integers.
The statement “$d$ divides $n$” means (in this context)
   there exists an integer $q$ such that $n = qd$.
A standard notation for this is $d \mid n$.
Notice that this defines a relation on the integers.
(For the case $d > 0$, “$d$ divides $n$” iff the remainder in the Division Algorithm (for $n$ divided by $d$) is
zero. But note that this definition of “divides” is completely independent of the Division Algorithm.)

(N.B. Note that the definition of “divides” contains no reference to the operation of division. Even
though the word “divides” is used, it is purely a multiplicative property.
The statement “$d$ divides $n$” means the same as each of the following:
   “$d$ is a factor of $n$”,
   “$n$ is a multiple of $d$”, and
   “$n$ is divisible by $d$.”)

BE SURE TO READ AND TO UNDERSTAND THE PRECEDING DEFINITION, and the other
words just given which are used to describe the relation! Of course, you should USE this definition in
your proofs. Use THIS definition. For example, “$n$ is divisible by $d$” does NOT mean (according to
the definition) that $n/d$ is an integer. (If we are in the world of integers, there is no division operation.)
It means, as stated above, there is an integer $q$ such that $n = qd$.

Note that if we look at the special case $p = 2$, we see that:
   An integer is even if and only if it is divisible by 2.

This definition defines a relation on the set of all integers (denoted by $|$) and is appropriate if we
know what integers are. If we’re still living in the world of natural numbers, as in Chapters 1 and 2 of
the textbook, and as in the early grades of school, then we restrict all variables to be natural numbers. If
it is necessary to make this distinction, we refer to “divisibility on the natural numbers”.