For all real numbers $a$ and $b$, the following intervals are defined. The notation on the LEFT is called “interval notation”. The set on the RIGHT is the DEFINITION of the interval on the left, using the usual set-builder notation. (See Additional Notes on notation below).

$$[a, b] = \{ x \in \mathbb{R} \mid a \leq x \leq b \}$$

$$(a, b] = \{ x \in \mathbb{R} \mid a < x \leq b \}$$

$$[a, b) = \{ x \in \mathbb{R} \mid a \leq x < b \}$$

$$(a, b) = \{ x \in \mathbb{R} \mid a < x < b \}$$

$$[a, \infty) = \{ x \in \mathbb{R} \mid x \geq a \}$$

$$(a, \infty) = \{ x \in \mathbb{R} \mid x > a \}$$

$$(-\infty, b] = \{ x \in \mathbb{R} \mid x \leq b \}$$

$$(-\infty, b) = \{ x \in \mathbb{R} \mid x < b \}$$

Each of the real numbers $a$ and $b$ is called an endpoint of the respective interval. Intervals which contain all of their endpoints $- [a, b], [a, \infty),$ and $(\infty, b]$ — are called closed, while those which contain none of their endpoints are open.

Additional Notes.

For the first four definitions above, it is often (but not always) assumed at least that $a \leq b$, sometimes, more restrictively, that $a < b$.

In this class this semester, we will always assume, in these cases, that $a$ and $b$ are chosen so that the interval defined is not empty. (Reference added April 5, 2021: This is more or less consistent with the discussion in Section 9.4.2 in the Madden/Aubrey textbook; see the first page of this Section and the first 12 lines of the second page). On occasion, we may assume more.

A basic property of an interval, one which could be used to define “interval” in general, is:

A set $J$ is an interval iff

If $a$ and $b$ are elements of $J$, then every real number between $a$ and $b$ is in $J$.

As usual, this last statement could also be expressed as

For all $a$ and $b$ which are elements of $J$, every real number between $a$ and $b$ is in $J$.

Note that this property, as stated, would include a set containing only one point as an interval, and the empty set as an interval. If you don’t want these sets to be intervals, you have to change the description of this property as just given here by ruling them out.

This definition also implies that the set $\mathbb{R}$ of all real numbers is an interval, often denoted by $(-\infty, \infty)$. 