**Geometric Series and Decimal Expansions**

1. **Background:** Review the formula for the sum of an infinite geometric series (from a previous course or from a reference). Below, you will be asked to write it down. (In an induction proof earlier in this class, we discussed the formula for the sum of a finite geometric series.)

   The formula for the sum of an infinite geometric series (such a series will henceforth, in this Lesson, be referred to simply as a “geometric series”) depends on two “parameters” \( a \) and \( r \):
   - The first term of the series, which means exactly what it says -- except, to be more precise, we might say “the first nonzero term” -- is frequently denoted by \( a \), and
   - the common ratio, or common multiple, which is commonly denoted by \( r \); for the series to converge (whatever that means), we must have \( |r| < 1 \).

   (a) **Give the formula for the sum of a geometric series** with first term \( a \) and common ratio \( r \).

   (b) **Explain** why the following is a geometric series, **identify** the first term and the common ratio, and **give** the sum of the series, using the "well-known" formula referred to above.

   \[ \frac{9}{10} + \frac{9}{10^2} + \frac{9}{10^3} + \ldots \]

   (c) **Write** the geometric series just given (not the simple number which is the sum you found above, but the given series) as a **decimal**, using the usual base-10 decimal notation.

   (Recall, e.g., that 3.14159 means
   \[ 3 + \frac{1}{10} + \frac{4}{10^2} + \frac{1}{10^3} + \frac{5}{10^4} + \frac{9}{10^5}. \]
   **THIS IS A REMINDER OF DECIMAL NOTATION.** This is not a geometric series!)

   (d) **Important question:** Is 0.999999... an element of \((0, 1)\)?

   [Maybe it’s that long sought after “largest element” of \((0, 1)\)?]

2. **These problems are not specifically about geometric series, and you don’t need the formula** for the sum of a geometric series anymore. There are reminders about decimal notation.

   (a) Use the Preceding Result(s) to give another, simpler decimal expression for 0.00999999... .

   (b) **Recall the difference between terminating decimals, such as 0.25, and nonterminating decimals, such as 0.333333... and 0.14159....**

   Write 0.25 as a nonterminating decimal. (I.e., find a nonterminating decimal which is equal to 0.25).