Refer to the notes from April 19 on the class website on the Density of Rationals (under “Notes from online class”), and prove the following, using the ideas in the notes:

1. Prove: If \( a < 0 < b \), then there exists a rational number \( r \) with \( a < r < b \).

2. If \( 0 \leq a < b \), explain why there exists a natural number \( m \) such that \( m > 1/(b - a) \).

3. Using the information and the ideas given in the notes, prove, using Problem 2 above, that if \( 0 \leq a < b \), then there exists a rational number \( r \) with \( a < r < b \).

(Problems 1 and 3 prove the density property for the two cases (I) when \( a \) and \( b \) are separated by 0, and (II) when both \( a \) and \( b \) are to the right of 0.)

4. Prove: If \( a < b \leq 0 \), then there is a rational number \( r \) with \( a < r < b \), using the ideas in the notes and previous results (such as 3 above).

Note Well. These problems are the main steps in our PROOF of the density of the rationals. Don’t USE the general idea of the density of the rationals to prove them. After completing Question 4 above and combining the results together, you will THEN have a proof of the density of the rationals, which of course can be used in future work.