A summary of basic high school algebra properties of real numbers which we assume:

Refer to Definition 15.1.1 at the beginning of Chapter 15 in the textbook. This defines an “ordered field”, but the point here is not the definition of an “ordered field”. The point is that the properties listed are all familiar properties of the real numbers, and we use them, usually without comment, in this class when dealing with examples involving real numbers. Assumed: The set of real numbers is an ordered field.

Some of the terms used might not be familiar, and they will be commented on here; but the properties themselves should all be familiar from high school algebra:

(In the discussion below, the word “number” is synonymous with “real number”.)

1. “total order”. Here, this refers to the usual order, <, on numbers, and you are familiar with the meaning of “a < b” for two numbers a and b. The idea that this is a “total” “order” summarizes the following two properties of <:
   • The usual basic order property, transitive: If a < b and b < c, then a < c.
   • The trichotomy property: Given numbers a and b, exactly one of the following is true: a < b, a = b, b < a. (The fact that we can use this order to “compare” any two numbers makes it a “total” order.)

2. “binary operation” refers to an operation on numbers a and b which gives another number; especially: addition, multiplication (also, subtraction, exponentiation, ... ). Item 2 specifically refers to addition, denoted by +. (Item 8 refers to multiplication.)

You should be familiar with the properties of “commutative”, “associative”, and “distributive”; if not, look them up.

The ideas of “additive identity” (the number 0) and “multiplicative identity” (the number 1) should be familiar, and similar ideas are discussed in linear algebra for vectors (the zero vector) and for matrices and linear transformations (e.g., the zero matrix and the identity matrix).

Similarly for “additive inverse” (−a is the additive inverse of a) and “multiplicative inverse” (a⁻¹ is the multiplicative inverse of a; of course, here a can’t be 0). Again, similar ideas are discussed in linear algebra.

As stated, you can assume these properties for real numbers in the examples that we do and you are asked to do, and usually you don’t need to refer to them by name. (E.g., when dealing with (a + b)/2, it makes no difference if you write this as (b + a)/2, or (1/2)(a + b), or (1/2)a + (1/2)b, etc. But it is good practice to think about what properties we are using, and what properties we used to prove that a < (a + b)/2 < b if a < b.)