All the problems on this exam are in the “world of real numbers”. Unless otherwise indicated, SETS (such as $S$ and $T$) ARE ASSUMED TO BE SETS OF REAL NUMBERS, and lower-case “variables” such as $x$ and $b$ are assumed to be real numbers.

5. Suppose a set $S$ has the following property:

For every natural number $n$, there exists $s$ in $S$ such that $s > n$.

Prove, using tools from this class and definitions given here (e.g., Problem 3), that $S$ is unbounded above.

**SOLUTION.** Proof. Suppose $S$ is a set with the property given.
Consider a real number $b$. By the Archimedean property, there is a natural number $n > b$.
By our assumption on $S$, there exists $s$ in $S$ such that $s > n > b$.
Thus, for every real number $b$, there exists an element $s$ in $S$ with $s > b$, and therefore by Problem 3 above, $S$ is unbounded above.