6. For the purposes of this exam, we give the following definition:
   Given a real number \( b \), we say that a set \( S \) of real numbers is \( b \)-bounded iff for all \( x \) in \( S \), \( x \leq b \).

Let \( b \) be a real number.
Prove or disprove, carefully and completely:
A set \( S \) of real numbers is \( b \)-bounded iff \( S = (-\infty, b] \).

SOLUTION. This statement is false. One can give a “general” counterexample, i.e., show that for every real number \( b \), the given statement if false, or, one can give a specific counterexample, showing that there exists a \( b \) such that the given statement is false.
We will give a general proof.

Let \( b \) be a real number.
Consider the set \( S = \{b\} \).
Consider \( x \) in \( S \). Then \( x = b \), so \( x \leq b \). So, by definition, \( S \) is \( b \)-bounded.
But clearly \( S \neq (-\infty, b] \). (E.g., \( b - 1 \) is an element of \( (-\infty, b] \), but not an element of \( \{b\} \).)

A specific counterexample had be obtained by doing the same thing for your specific favorite real number \( b \).