8. The Avogadro-Archimedean property is:
   (AAP) For every real number $x > A$, there exists a natural number $n$ such that $n > x$.

   (b) Prove that (AAP) is equivalent to the Archimedean property,
       (AP) For every real number $x$, there exists a natural number $n$ such that $n > x$.

   Even though part or all of this proof is very simple, give the proof using the standard approaches discussed in class for proving such statements; one of the purposes of this problem is to test whether you can give proofs in this form, since it is usually the best way to approach such proofs, especially more complicated proofs.

SOLUTION.
First, we will assume (AAP) and prove (AP):
   (AP) For every real number $x$, there exists a natural number $n$ such that $n > x$.

   How Do You Start Proof?
   Assume $x$ is a real number. We consider two cases:
   Case 1. Suppose $x > A$. Then, by (APP), there exists a natural number greater than $x$.
   Case 2. Suppose $x \leq A$. Then $A + 1 > A \geq x$, so $A + 1$ is a natural number greater than $x$.
   [Note how simple this Case 2 is.]

   This prove (AP).

   Conversely, we will assume (AP) and prove (AAP):
   (AAP) For every real number $x > A$, there exists a natural number $n$ such that $n > x$.

   How Do You Start Proof?
   Assume $x$ is a real number $> A$. By (AP), we can find a natural number $> x$.
   This prove (AAP).

We have shown that (AAP) implies (AP), and (AP) implies (AAP). Thus, these two statements are equivalent.