Suppose \( n \) is a natural number. I.e., throughout the discussion below, let \( n \) be a “fixed but arbitrary” natural number.

Consider the statement

“Every set of real numbers with exactly \( n \) elements has a least element”.

(Note that the words “every set of real numbers” means exactly the same as “every subset of the set of real numbers”.

Rewrite this statement in two different, equivalent ways:

First, by using quantifiers, “for all” and “there exists”, with the usual notation for quantifiers.

Second, as an “if-then” statement.

You will have to (i.e., you should) introduce at least one “variable” to do this.

**SOLUTION using quantifiers.**

\[ \forall S \in P(\mathbb{R}) \text{ with exactly } n \text{ elements, } \exists x \text{ such that } x \text{ is the least element of } S. \]

[As usual, there are many other ways to say this. For this example, the original “Every set ... version, or one of the following “if-then” statements, may be the best way:]

**SOLUTION using “if-then”.

If a subset of \( \mathbb{R} \) has exactly \( n \) elements, then the subset has a least element.

or, with a variable ...

If \( S \in P(\mathbb{R}) \) and \( S \) has exactly \( n \) elements, then \( S \) has a least element.

Continue to assume, as above, that \( n \) is a natural number.

**SOLUTION.**

Assume \( S \) is a subset of \( \mathbb{R} \) with exactly \( n \) elements.

or

Assume \( S \in P(\mathbb{R}) \) and \( S \) has exactly \( n \) elements.

[Remember, in general, one of the points of this initial statement is to introduce the names of the “variables” you will be using in the proof. For completing the proof, it’s very useful, as usual, to give the set you’re talking about a name, such as \( S \) or \( A \) or ... . It’s not necessary to include this in the first statement, but it may be the most convenient thing to do. Other acceptable alternatives:] Consider a set \( S \in P(\mathbb{R}) \) with exactly \( n \) elements.

Consider a subset of \( \mathbb{R} \) with exactly \( n \) elements. Let \( S \) be such a set.