Example from HW 18.

Let $D = \{4, 5, 6\}$.

What is $\text{UB}(D)$?

Claim: $\text{UB}(D) = [6, \infty)$ [use this notation instead of $\{x : x \geq 6\}$, and no need to give another name to this set, unless it is really easier to write].

Proof.

Suppose $b$ is an element of $[6, \infty)$. Then $6 \leq b$.

[To prove: $b$ is an element of $\text{UB}(D)$; i.e., to prove $b$ is an upper bound of $D$; i.e., to prove for all $x$ in $D$, $b \geq x$.]

Suppose $x$ is an element of $D$. Then $x$ is 4 or 5 or 6. Each of these is $\leq 6$, so $x \leq 6 \leq b$.

Thus, by definition of upper bound, $b$ is an upper bound of $D$, so $b$ is an element of $\text{UB}(D)$.

Conversely, suppose $b$ is an element of $\text{UB}(D)$.

Then, by definition of $\text{UB}$, for all $x$ in $D$, $b \geq x$.

[To prove: $b$ is an element of $[6, \infty)$.]

Since 6 is an element of $D$, $b \geq 6$.

Thus, $b$ is an element of $[6, \infty)$.

[Note how we use the universally quantified statement]

[Note that we couldn't do this if 6 were not in the set; proof would be more complicated if 6 was not in the set $D$, and might want to use contrapositive: if $b < 6$, then $b$ is not an element of $\text{UB}(D)$.]

Thus, $b$ is an element of $\text{UB}(D)$ iff $b$ is an element of $[6, \infty)$.

So, $\text{UB}(D) = [6, \infty)$. 