Solutions to Problems for appendix B

 # 2 p. 618 $\boxed{-5 = 5e^{i\pi}}$ since $e^{i\pi} = \cos(\pi) + i\sin(\pi) = -1 + i \cdot 0 = -1$

So $r = 5$ and $\theta = \pi$.

 # 6 p. 618 $-i = 0 + i(-1) = \boxed{e^{-i\frac{\pi}{2}} = -i}$ since $\cos(-\frac{\pi}{2}) = 0$ and $\sin(-\frac{\pi}{2}) = -1$.

So $r = 1$ and $\theta = -\frac{\pi}{2}$ (or $\theta = \frac{3\pi}{2}$).

 # 8 p. 618 $z = 5 - 12i$ $|z|^2 = 5^2 + 12^2 = 25 + 144 = 169 = 13^2$

so $r = |z| = 13$

$\tan\theta = \frac{-12}{5}$ i.e. $\theta = \arctan\left(\frac{-12}{5}\right) \simeq -1.176$ (since z is in the 4^{th} quadrant).

So $\boxed{5 - 12i = 13e^{i\theta} \text{ with } \theta = \arctan(\tfrac{-12}{5})}$.

 # 12 p. 618 $(1+i)^2 + (1+i) = 1 + 2i + i^2 + 1 + i = 1 + 2i - 1 + 1 + i = \boxed{1 + 3i = (1+i)^2 + (1+i)}$.

 # 15 p. 619 $\left(e^{i\frac{\pi}{3}}\right)^2 = e^{i\frac{2\pi}{3}} = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right) = \boxed{-\frac{1}{2} + i\frac{\sqrt{3}}{2} = \left(e^{i\frac{\pi}{3}}\right)^2}$.

 # 18 p. 619 $\sqrt[4]{10e^{i\pi/2}} = \sqrt[4]{10}\left(e^{i\pi/2}\right)^{1/4} = 10^{1/4}e^{i\pi/8} = 10^{1/4}\cos\left(\frac{\pi}{8}\right) + i\,10^{1/4}\sin\left(\frac{\pi}{8}\right)$

Note that there a $\underline{\underline{4}}$ fourth-roots of $10e^{i\pi/2}$ (and of any complex number).

 # 23 p. 619 $(1+i)^{100} = \left[\sqrt{2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)\right]^{100} = \left(\sqrt{2}\,e^{i\pi/4}\right)^{100} = 2^{50}e^{25i\pi}$

$= 2^{50}e^{24i\pi}e^{i\pi}$ Since $e^{24i\pi} = 1$ and $e^{i\pi} = -1$

we have $\boxed{(1+i)^{100} = -2^{50}}$.

 # 26 p. 619

Let $z = \sqrt{3} + i$. $|z|^2 = 3 + 1 = 4$ so $|z| = 2$

$\tan\theta = \frac{1}{\sqrt{3}} = \frac{1/2}{\sqrt{3}/2} = \tan\left(\frac{\pi}{6}\right)$, so $\theta = \frac{\pi}{6}$

Therefore, we can write $\sqrt{3} + i$ as $\sqrt{3} + i = 2e^{i\frac{\pi}{6}}$.

Thus, $(\sqrt{3}+i)^{1/2} = \pm\left(2e^{i\pi/6}\right)^{1/2} = \pm\sqrt{2}\, e^{i\pi/12} = \pm\sqrt{2}\left[\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{12}\right)\right]$

i.e. $\boxed{(\sqrt{3}+i)^{1/2} \simeq \pm(1.366 + i\, 0.366)}$.

38 p. 619 $\boxed{\text{True}}$

Let $z = x+iy$ be an arbitrary complex number.

$z\bar{z} = x^2+y^2$ is a real number since x and y are real.

39 p. 619 $\boxed{\text{False}}$

Let $z = x+iy$ be an arbitrary complex number.

$z^2 = x^2 + 2xy + (iy)^2 = x^2 - y^2 + i\,(2xy)$

So as long as x and y are not both 0, z^2 will not be real.

For instance, let $z = 1+i$; $z^2 = 2i$ is not real.

41 p. 619 $\boxed{\text{True}}$

We know every complex number z can be written in the form $re^{i\theta}$, where $r>0$ if $z \neq 0$.

Therefore, since $r = e^{\ln(r)}$, we have $z = e^{\ln(r)}e^{i\theta}$, i.e. $z = e^{w}$ with $w = \ln(r) + i\theta$.