## The Physics of Networks: An Analysis

This paper looks at and analyzes the issue of networks and their properties from a more analytical point of view than that which has been used previously. A network is a group of nodes that are all interconnected. It is a form of model for complex systems which can be used in many different types of situations. Essentially, the nodes can be anything, as long as there is some sort of rule which dictates the nature of their connection. Networks have been studied at a basic level as early as 1736 by Leonhard Euler in his solution to the Seven Bridges of Königsberg. In the past, however, networks have not been used very much as models to represent something more complex, like this paper does, but rather social networks are what were most often studied in order to determine the implications of the properties of a network for that community. There has been some limited analysis based on the mathematical properties of existing networks, as previously mentioned, but it took physicists to bring this idea to the next level and use networks as a working model for many things in order to predict how the system will behave. This type of analysis is based on the philosophy of physics, which is being applied to something which is clearly not physical in nature.

There are several important properties of networks which are used by physicists in analyzing a network, and some of these are: node degree, degree distribution, and the correlation of a network. The node degree is the number of edges coming from a single node. If it has many edges or connections, it will have a high degree. These high-degree nodes are known as hubs. If it has few connections, it is a low degree node. The way these degrees are distributed throughout the network is a very important point to consider, especially when

looking at a network such as the internet or disease spread where the removal of a node can potentially have a great effect on the rest of the network. There are essentially two types of distributions examined in this paper because they are the most common in a network situation, and these are: a scale-free distribution, which is based on a power law, giving it a large amount of low-degree nodes and a very small amount of high-degree nodes; and a Poisson distribution, which is a random distribution with most of the nodes having degree values close to the average with very few in either extreme. In the case of the internet, a scale-free distribution would be more resilient to hardware failure or some other non-targeted failure since the majority of its nodes are still low-degree. However, if there were a targeted attack, since there are so few hubs, all of the other nodes are that much more reliant on them and taking one out could be disastrous for the whole network. A Poisson distribution is guite the opposite – a targeted attack would not be very effective since most nodes are near the average and thus are not very reliant on a small number of nodes, but random failure would likely be more damaging. It turns out that most of the internet has a scale-free distribution. Thus, it is very important that the few hubs should be well-protected. Or in the case of disease spread, a scalefree distribution would keep the disease from spreading very much unless it got to one of the hubs, and then it would spread very quickly. A Poisson distribution would have an average rate of spread all the way through. Analysis of these networks allows us to see these things and act upon them, even if we can't fully understand such complex systems.

The correlation of a network is also an important property. This basically looks at the way nodes are connected to other nodes. If a network is positively correlated, this means that

the nodes that high-degree nodes are connected to have, on average, a higher degree than the rest of the nodes, and the nodes that the low-degree nodes are connected to have a lower average than the rest of the nodes. This is called a core-periphery structure because there is a core of hubs at the center of the network which is surrounded by a periphery of low-degree nodes. If it is negatively correlated, the high-degree nodes are connected to mostly lower degree nodes and vice-versa. This just means that the hubs are more spread out and connect to more nodes with different degrees. In general, social networks are positively correlated, suggesting that popular people are more likely to have friends that are relatively popular as well, and unpopular people are not as likely to be friends with the popular people, but nonsocial networks such as the internet are negatively correlated. This property can be used also to predict the behavior of a network. In a disease network, for example, if it is positively correlated, then it is easy for the disease to persist if it gets to the core because of the high degree density there, which makes it easy to be passed on. However, it is not as likely that it will spread to the periphery because of the low density there. But if the network is negatively correlated, then it is more difficult for the disease to persist because it can easily be passed to a low-degree node and then not passed on after that. If it does persist, though, it has the potential to infect the whole network. Thus, it is clear that properties such as this can be used to generalize the behaviors of a networked system, even if the intricacies of the network are too complex to understand. This is, in fact, the whole idea behind a model - to take something seemingly impossibly complex, and simplify it in a way that allows us to understand it.

Many experiments have been done concerning the properties of networks, and one of the most well-known of these is the "six degrees of separation" or the "small world effect". This is the idea that everyone in the world is connected to everyone else in very few steps. The first main experiment examining this idea was done at Harvard by Stanley Milgram. The experiment was conducted such that a person would get a letter and be instructed to deliver this letter to a specified person who was unknown to the individual, but only by giving it to someone he/she knew personally. It was discovered that the letter would, on average, consistently reach its target in a surprisingly small number of steps. This essentially says that, in a networked system, everything, or everyone in this case, is connected to everything else in a small number of steps. This can be represented by the mathematical equation  $k^n = p$  where k is the number of degrees of any one node, n is the number of steps it takes to get to any other target, and p is the population, or the number of nodes in the network. When it is observed that it is an exponential function, it is not quite so surprising that this is true, since even with a small number of friends, by the time it approaches the sixth power, it will encompass a colossal amount of people. These mathematical advances in network properties were crucial and opened the way for more research. Another interesting thing that was first noticed by Jon Kleinburg is that, though the people in the experiment could not observe the whole network in order to make the best choices, the letter always reached its destination in a very efficient manner (that is, with a small number of steps). This just shows how a member of a network can comprehend it on a basic level without being able to see the whole picture.

Another interesting experiment is the one about bottlenose dolphins. A program was used to analyze a community of dolphins based on the pattern of connections, and was able to separate out two distinct sub-groups within the network. Then, one of the dolphins left and this finally caused the dolphins to separate out into different groups. The computer program was actually fairly accurate in its prediction of the different groups. This type of analysis could be used in many network situations to predict its behavior. However, we come back to the shortcoming of every model, and that is that it makes assumptions. There were three dolphins that did not behave as predicted and went with the other group. Why should this be? They had more connections with that group, but yet they went with the other. Clearly, this mistake must be attributed to a complexity in the network which was overlooked by the model and the computer program, which shows that we don't yet understand all the intricacies involved in a network system.

As we further our understanding of networks, we can make better models, and therefore better predictions. This is what physicists brought to the problem. Their scientific way of thinking has allowed for a much better understanding of the way a network behaves in many different situations, not just in a social environment, which allows for application of the model to many other things. The act of looking at and quantifying different properties of networks allows us to see how such a system behaves and to predict its future behavior. As the scientific community learns from its mistakes, like with the dolphin problem, it will increase its understanding of the subject and advance the techniques for looking at a specific property or maybe even discover a new property, opening the door for better predictions and a better

understanding of networks as a whole. This spirit of scientific advancement is at the core of

Mark Newman's paper.