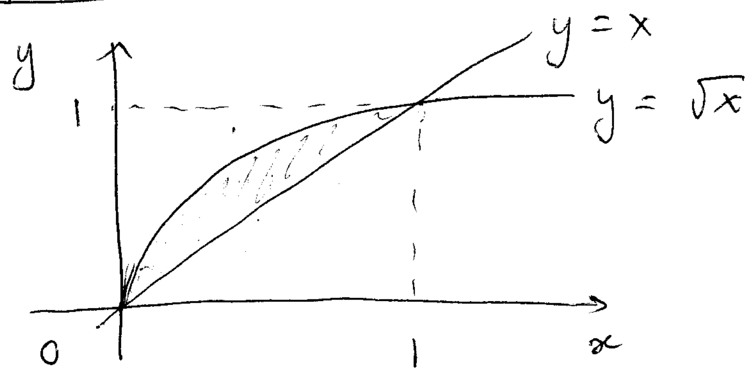
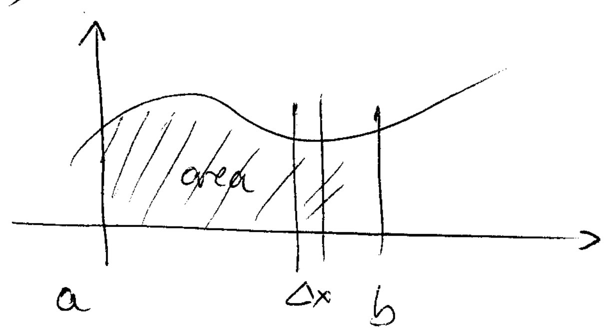
Area, volume, arc length, density & center of mass

1. Areas

Example 1:



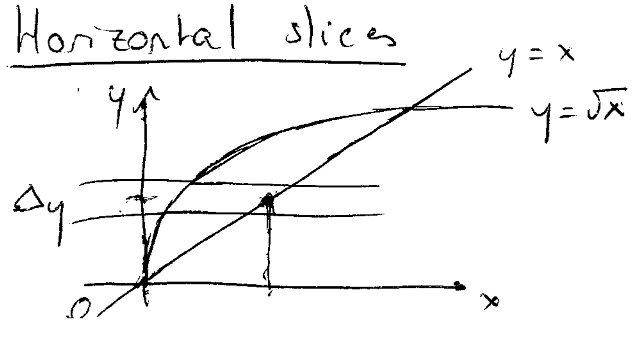
1. Area under curve $y = f(x)$ is $\int_a^b f(x)\,dx$
$(f(x) \geq 0)$



Riemann sum: $\sum_{i=1}^{N} f(x_i)\,\Delta x \qquad \Delta x = \frac{b-a}{N}$

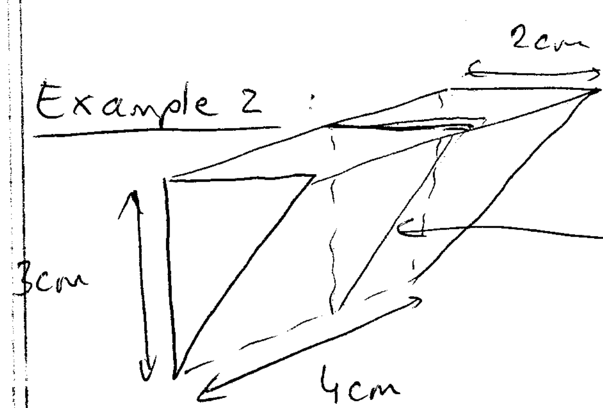
$$\lim_{\Delta x \to 0} \sum_{i=1}^{N} f(x_i)\,\Delta x = \int_a^b f(x)\,dx$$

Horizontal slices



$$A = \int_0^1 (y - y^2)\,dy$$
$$= \left[\frac{1}{2}y^2 - \frac{1}{3}y^3\right]_0^1 = \frac{1}{2} - \frac{1}{3}$$
$$= 1/6$$

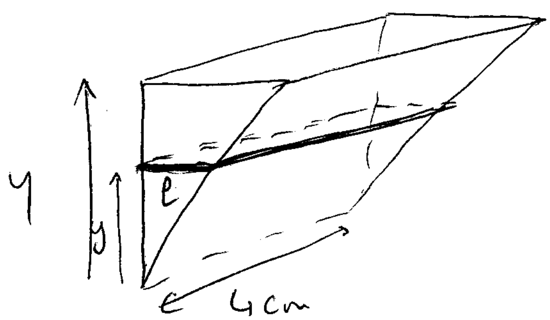
Example 2 :



slice has area: $\frac{3 \times 2}{2} = 3\ cm^2$

Volume $= 3 \times 4 = 12\ cm^3$

Now slice in the horizontal direction:



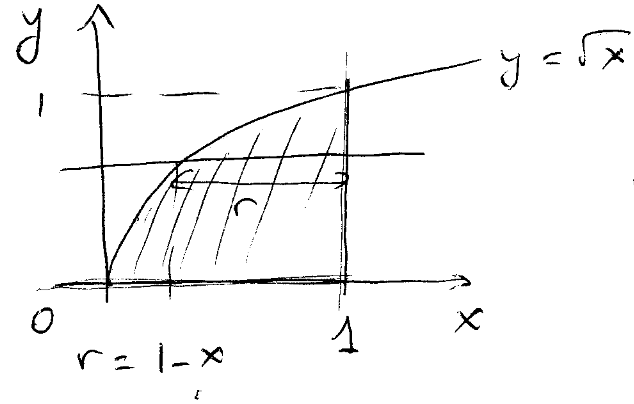
Each slice has thickness Δy

Area $= 4 \cdot l\ (cm^2)$

$$\frac{l}{y} = \frac{2}{3} \Rightarrow l = \frac{2}{3} y$$

$$V = \int_0^3 4 \cdot \frac{2}{3} y \, dy = \left[\frac{8}{3} \frac{y^2}{2}\right]_0^3 = \frac{4}{3} 3^2 = 12\ cm^3$$

8. Volumes



Use horizontal slices

$$V = \int_0^1 \pi (1 - y^2)^2 \, dy$$

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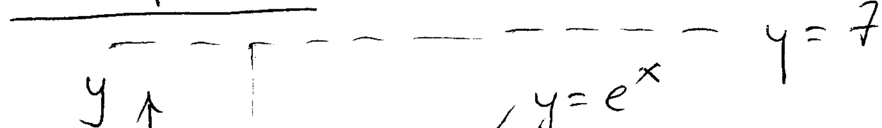
$$= \pi \int_0^1 (1 - 2y^2 + y^4) \, dy = \pi \left[y - \frac{2}{3} y^3 + \frac{1}{5} y^5 \right]_0^1$$

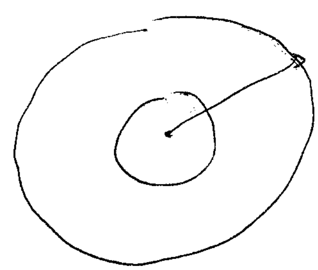
$$= \pi \left[1 - \frac{2}{3} + \frac{1}{5} \right] = \pi \, \frac{15 - 10 + 3}{15} = \pi \, \frac{8}{15} \simeq 1.68$$

Example 2 :



Use vertical slices

Area of each slice
$A = \pi \left(7^2 - (7 - e^x)^2\right)$



$$V = \int_0^1 \pi \left(7^2 - (7 - e^x)^2\right) dx$$