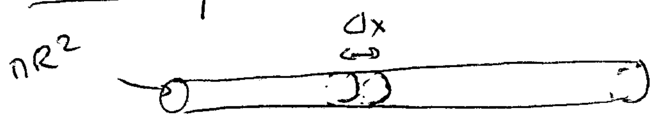
Area, volume, arc length, density, & center of mass (continued)

4. Density & center of mass



of slice

mass $= \Delta m$

volume $= \pi R^2 \Delta x$

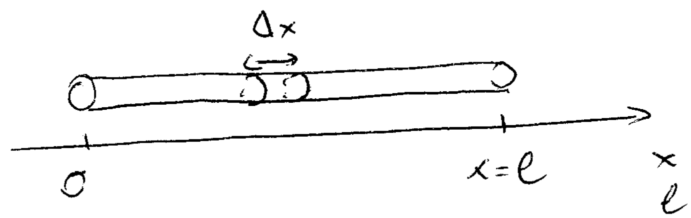
$$\text{density} = \delta(x) = \lim_{\Delta x \to 0} \frac{\Delta m}{\pi R^2 \Delta x}$$

In one dimension, $\tilde{\delta}$ has units of $\frac{\text{mass}}{\text{length}}$

density

In 2 dimensions, $\tilde{\delta}$ would have units of $\frac{\text{mass}}{\text{area}}$

In 1 d, $\tilde{\delta} = \pi R^2 \delta$



$\delta(x) =$ density

$$m = \text{mass of rod} = \int_0^{\ell} \delta(x)\,dx$$

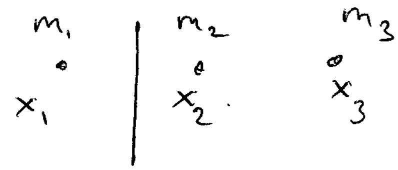
Slice of length Δx has mass $\Delta m = \Delta x\, \delta(x)$

$$m = \lim_{\Delta x \to 0} \sum_{i=1}^{n} \Delta x\, \delta(x_i) = \int_0^{\ell} \delta(x)\,dx$$

Example : $\delta(x) = e^{-x}$ g/cm $\quad \ell = 10$

$$m = \int_0^{10} \underbrace{e^{-x}}_{\text{in g/cm}} \underbrace{dx}_{\text{cm}} = \left[-e^{-x}\right]_0^{10} = (1 - e^{-10}) \text{ grams}$$

Center of mass

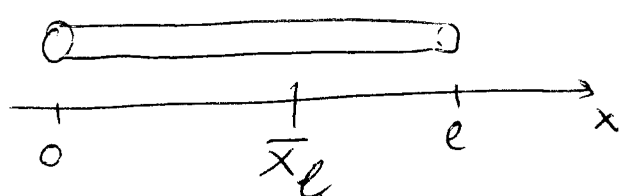


$$\bar{x} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$\bar{x}$ = x-coordinate of center of mass

For a continuous rod:

$\delta(x)$ = density of rod (in $\frac{\text{units of mass}}{\text{units of length}}$



$$\bar{x} = \frac{\int_0^{\ell} x\,\delta(x)\,dx}{m}$$