Sequence and series

1. Sequences

Example 1: $s_n = (-1)^n \left(\frac{1}{3}\right)^n$

$n=1 \quad s_1 = \frac{-1}{3} \qquad n=2 \quad s_2 = \frac{1}{9}$

$n=3 \quad s_3 = \frac{-1}{27} \qquad n=4 \quad s_4 = \frac{1}{81}$

Example 2: $s_1 = 0$; $s_2 = 2$; $s_3 = 6$; $s_4 = 14$; $s_5 = 30$

$s_{n+1} = 2 s_n + 2 \qquad s_n = 2^n - 2$

Example 3:

$$\begin{aligned} s_1 &= 1 \\ s_2 &= 2 s_1 + 3 \\ s_3 &= 2 s_2 + 3 \\ s_4 &= 2 s_3 + 3 \end{aligned}$$

$s_k = 2 s_{k-1} + 3 \qquad k > 1$

$$2^{n+1-k} s_k = 2^{n+1-k} (2 s_{k-1} + 3)$$

$$\begin{aligned} \sum_{k=2}^{n+1} 2^{n+1-k} s_k &= \sum_{k=2}^{n+1} 2^{n+1-k} (2 s_{k-1} + 3) \\ &= \sum_{k=2}^{n+1} \left(2^{n+2-k} s_{k-1} + 3 \cdot 2^{n+1-k}\right) \\ &= \sum_{j=1}^{n} \left(2^{n+2-j-1} s_j + \frac{3}{2} 2^{n+2-j-1}\right) \qquad j = k-1 \\ &= \sum^{n} \left(2^{n+1-j} s_j + \frac{3}{2} 2^{n+1-j}\right) \end{aligned}$$

$$\sum_{k=2}^{n+1} 2^{n+1-k} s_k - \sum_{j=1}^{n} 2^{n+1-j} s_j = \sum_{j=1}^{n} \frac{3}{2}\, 2^{n+2-j}$$

$$s_{n+1} - 2^n s_1 = \frac{3}{2} \sum_{m=1}^{n} 2^m \qquad m = n+2-j$$

$$s_{n+1} = 2^n s_1 + \frac{3}{2} \sum_{m=1}^{n} 2^m = 2^n + \frac{3}{2} \sum_{m=1}^{n} 2^m$$

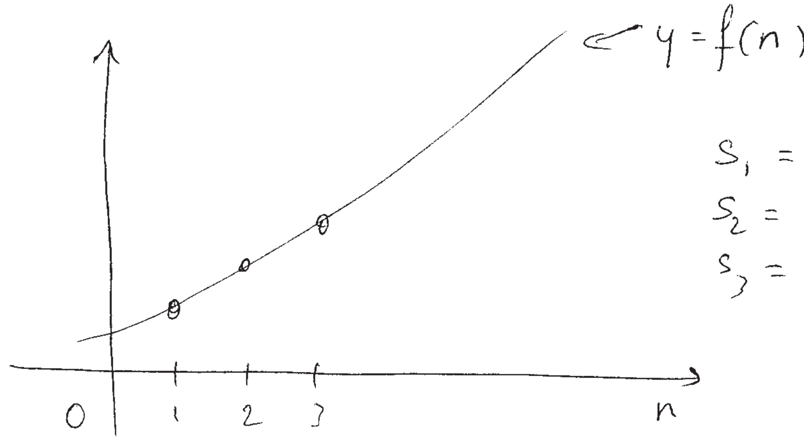
$$(2) \quad \sum_{m=1}^{n} 2^m = 2 + 2^2 + 2^3 + 2^4 + \dots + 2^n$$

$$(-)$$

$$(1) \quad 2 \sum_{m=1}^{n} 2^m = 2^2 + 2^3 + 2^4 + \dots + 2^n + 2^{n+1}$$

$$\sum_{m=1}^{n} 2^n = 2^{n+1} - 2 = 2(2^n - 1)$$

So $\quad s_{n+1} = 2^n + 3 \cdot (2^n - 1) = 2^n + 3.2^n - 3$



$s_1 = f(1)$
$s_2 = f(2)$
$s_3 = f(3)$