2/11/2009

Sequences & series (continued)

1. Sequences (continued)

Example 3: $\lim_{n\to\infty}\left(\frac{2^n}{5^n}\right) = \lim_{n\to\infty}\left(\frac{2}{5}\right)^n = 0$

So $\left(\frac{2^n}{5^n}\right)$ is a convergent sequence, and it converges to 0.

Example 4: $\lim_{n\to\infty}\left(\frac{n}{2} + \frac{5}{n}\right) = +\infty$

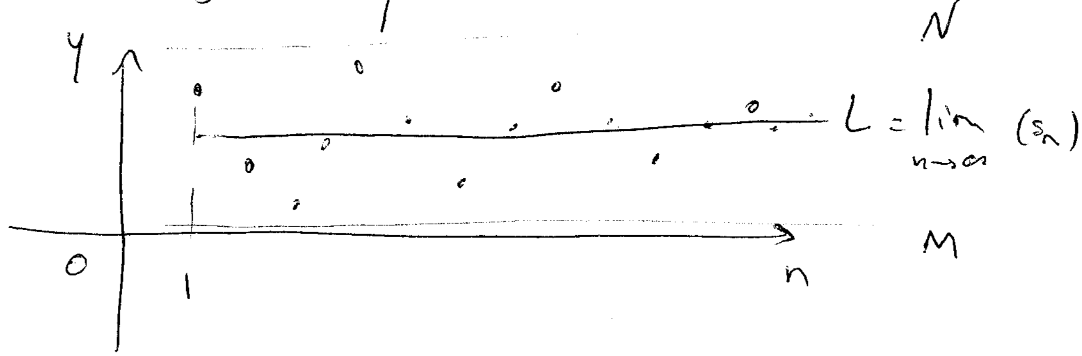
This sequence diverges.

Example 5: $\lim_{n\to\infty} \frac{\sin(2n)}{n} = 0$

So the sequence $\left(\frac{\sin(2n)}{n}\right)$ converges to 0.

Remarks:

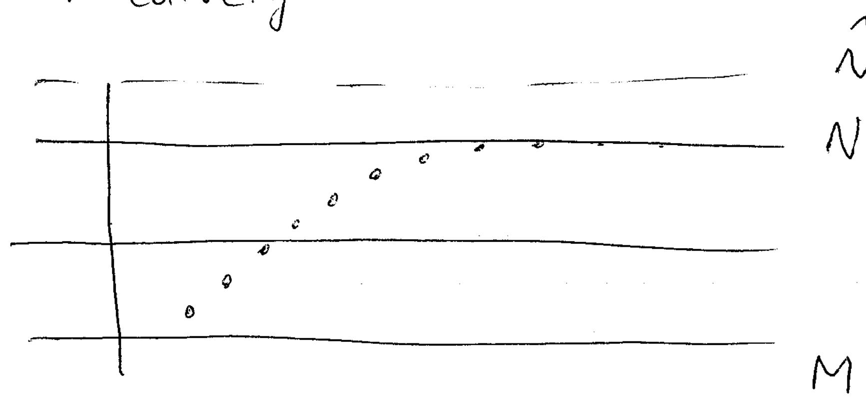
1. A convergent sequence is bounded



eg. $0 < \left(\frac{2}{5}\right)^n \leq \frac{2}{5}$ for $n \geq 1$

$-2 < \frac{\sin(2n)}{n} < 2$ for $n \geq 1$

2. If a sequence is bounded and monotone, then it converges.



<u>Counter-example</u>: $(\sin(2n))$ is bounded but does not converge.

2. <u>Series</u>:

A <u>series</u> is a pair of sequences (S_n) & (u_n) such that $S_n = \sum_{k=1}^{n} u_k$

A <u>geometric series</u> is of the form

$$S_n = \underbrace{a + ax + ax^2 + ax^3 + \dots + ax^{n-1}}_{n \text{ terms}}$$

Can we calculate S_n?

$$x S_n = \quad ax + ax^2 + \dots + ax^{n-1} + ax^n$$

$$S_n = a + ax + ax^2 + \dots + ax^{n-1}$$

$$(x-1) S_n = ax^n - a = a(x^n - 1)$$

So $S_n = a\dfrac{x^n - 1}{x - 1} = a\dfrac{1 - x^n}{1 - x}$ ← # of terms in series

1st term of series

multiplier

Example 1: Is $2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$ geometric

Yes $a = 2$ $x = \frac{1}{2}$