Sequences & series (continued)

2. Series (continued)

Recall:

If you have a sequence u_n, $n \geq 1$, then define the partial sums

$$S_n = \sum_{j=1}^{n} u_j$$

$S_1, S_2, S_3, \ldots$ forms a sequence.

You can think of a sequence as the "analog" of a function

$$u_n = f(n)$$

Series $\sum_{j=1}^{n} u_j = F(n)$ can be thought of as a "function" of n

There is an analogy between series & integrals

Convergence of a sequence as $n \to \infty$ $\Leftrightarrow$ convergence of a function as $x \to \infty$

Convergence of a series $\Leftrightarrow$ convergence of an improper integral of the form $\int^{\infty} f(x)\,dx$.

Example 2 : $1 + x + 2x^2 + 3x^3 + 4x^4 + \cdots$

Is this series geometric ?

No because the ratio of consecutive terms in the sum changes.

Example 3 : Find $3(0.1) + 3(0.1)^2 + 3(0.1)^3 + \cdots + 3(0.1)^{10}$

$$= 3\left[0.1 + (0.1)^2 + (0.1)^3 + \cdots + (0.1)^{10}\right]$$

$$= 3\left(0.1 \, \frac{1-(0.1)^{10}}{1-0.1}\right) \simeq 0.3333\ldots$$

Example 4 : If $\sum_{h=1}^{n} u_h$ is geometric, and if $u_h \neq 0$ for all h's, is $\sum_{h=1}^{n} \frac{1}{u_h}$ geometric ?

$$\sum_{h=1}^{n} u_h = a + ax + ax^2 + \cdots + ax^{n-1}$$

$$\sum_{h=1}^{n} (u_h)^{-1} = \frac{1}{a} + \frac{1}{a}\frac{1}{x} + \frac{1}{a}\left(\frac{1}{x}\right)^2 + \cdots + \frac{1}{a}\left(\frac{1}{x}\right)^{n-1}$$

So yes, $\sum_{h=1}^{n} \frac{1}{u_h}$ is a geometric series.

Convergence of series

Compare $\lim_{n\to\infty} \sum_{i=1}^{n} u_i$ with $\lim_{b\to\infty} \int_1^b f(x)\,dx$

(think of $u_i = f(i)$)