

Linear systems (continued)

1. Introduction (continued)

Example 1 (continued): $y'' - 5y' + 6y = 0$

2 distinct real eigenvalues: d_1, d_2

If $d_1, d_2 > 0$ origin is unstable

it is called a source or unstable node

If $d_1, d_2 < 0$ origin is stable

it is a sink or stable node

Example 2: $y'' + 9y = 0$

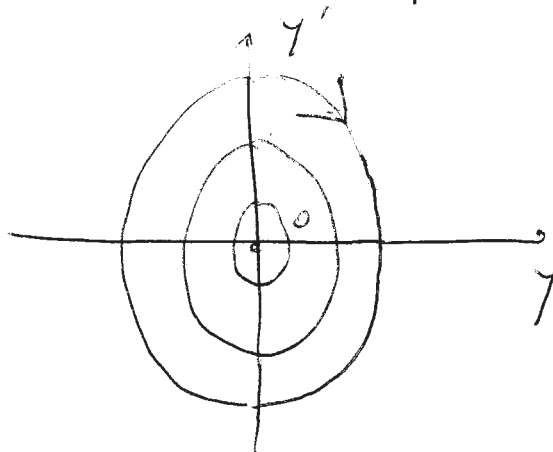
$$\lambda^2 + 9 = 0 \Leftrightarrow \lambda = \pm 3i$$

$$y(t) = C_1 \cos(3t) + C_2 \sin(3t)$$

$$y'(t) = -3C_1 \sin(3t) + 3C_2 \cos(3t)$$

Both y & y' are periodic functions \Rightarrow
trajectories are closed curves.

In fact, they are ellipses



The origin is stable
It is called a
center.

Example 3: $y'' - 6y' + 10y = 0$

$$d^2 - 6d + 10 = 0 \Leftrightarrow (d-3)^2 + 1 = 0$$

$$d = 3 \pm i$$

The origin is unstable. It is called an unstable spiral.

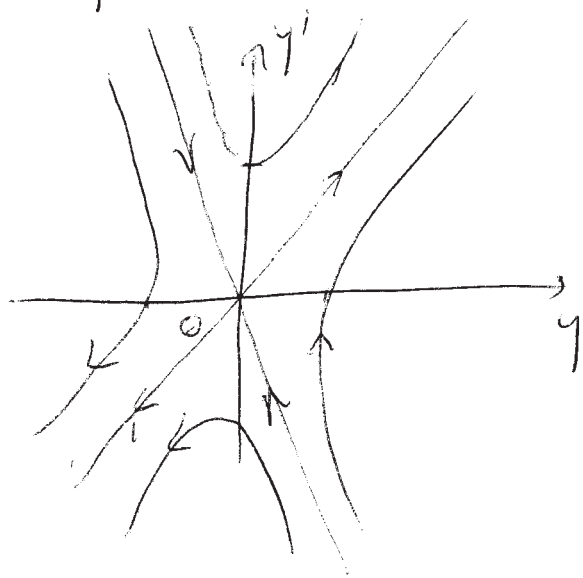
If the roots are of the form $d = \alpha \pm i\beta$,
 $\alpha > 0 \Rightarrow$ origin is an unstable spiral
 $\alpha < 0 \Rightarrow$ " a stable spiral

Example 4: $y'' + y' - 6y = 0$

$$d^2 + d - 6 = 0 \quad d = -3, 2$$

$$y = C_1 e^{-3t} + C_2 e^{2t}$$

$$y' = -3C_1 e^{-3t} + 2C_2 e^{2t}$$



The origin is unstable.
 It is called a saddle.