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In the Logistics Equations

Consider, $\frac{dN}{dt} = RN\left(1-\frac{N}{K}\right)$

however we will be using the descrete case

$x_{n+1} = r x_n (1-x_n)$ such that $x_n \geq 0$ and $r > 1$

a.) Find any fixed points of the logistics map, such that $\bar{x} = r\bar{x}(1-\bar{x})$

$$\frac{\bar{x} = r\bar{x}(1-\bar{x})}{r\bar{x}}$$

$= \frac{1}{r} = 1-\bar{x}$

$= -\frac{1}{r} = -1+\bar{x}$

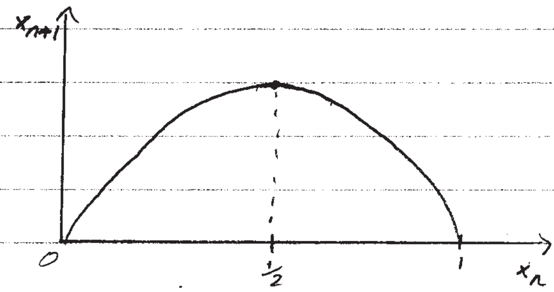
$= 1-\frac{1}{r} = \bar{x}$

so fixed points at

$\bar{x} = 1-\frac{1}{r}$ and we lost a solution at 0

$$\boxed{\bar{x} = 1-\frac{1}{r}, 0}$$

1. b. To determine the stability of the fixed points found above in part a, x_{n+1} is plotted against x_n:

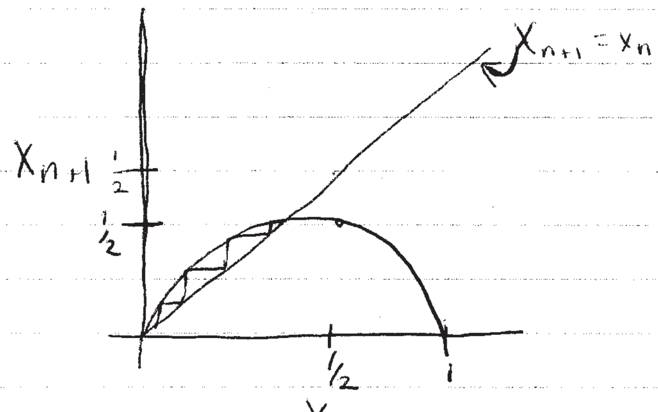


However, to maintain the condition $x_n \geq 0$ for any value r in the system $x_{n+1} = rx_n(1-x_n)$ certain constrains exist upon the system, specifically which values r can take.

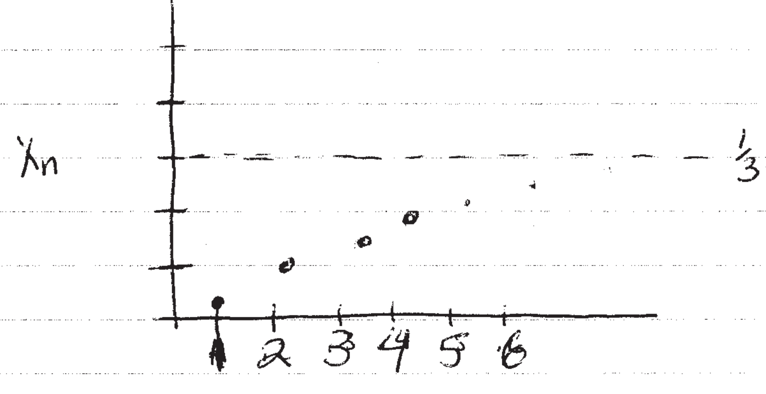
- It is given that $r > 1$
- $r \neq 0$
- r cannot be greater than 4, or system "blows up" upon x_{n+1}

$\therefore$ $1 < r \leq 4$

c.) Plot the case $r = 1.5$ and iterate the map starting from $x_1 = 1$



Iterate



This means the fixed points are stable at $r = 1.5$

$X_2 = .135$

$X_3 = .175$

$X_4 = .26$

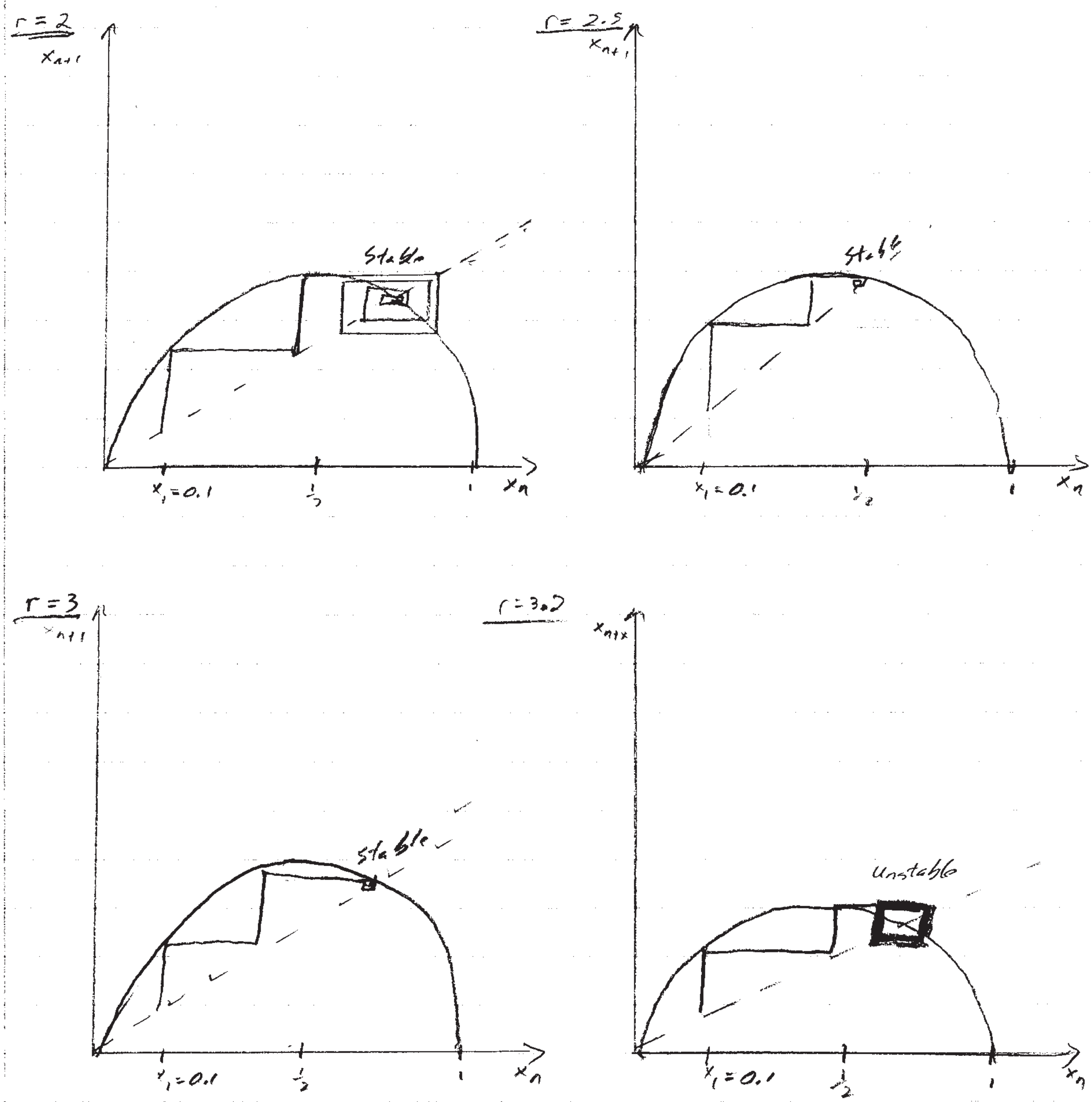
$X_5 = .354$

$X_6 = .284$

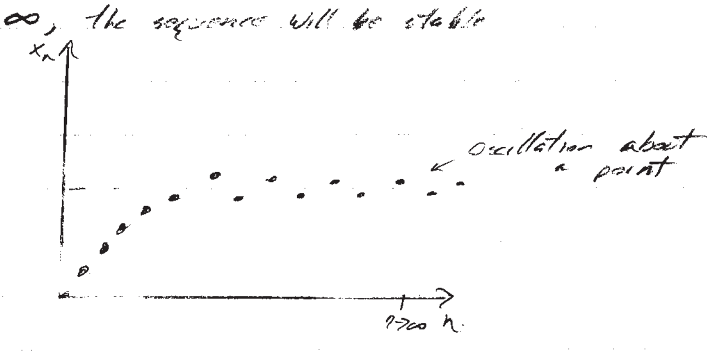
$X_7 = .305$

These both show stability at the fixed point! because the converge to a value.

1. a.



- As $r \geq 3.2$, the cobweb plot retraces itself and is unstable, but when $1 < r < 3.2$, the cobweb plot is stable about some point, though it does not converge to that point.
- This indicates that as $n \to \infty$, the sequence will be stable around some value but that it will also oscillate around that value.



e.) In this section we used the website to explore values numerically to examine and expriment with diffrent values of r.

We found that at $r \geq 3.2$ the ~~[illegible]~~ plots are nolonger stable. We get chaos!

If you want to check out the website it is listed on the problem page posted under the lecturenotes.

f.) Beyond values of $r = 3.57$ chaos occurs. now what happens if we change x_1 (the initial conditions) slightly. How does this differ from smaller values of r?

When we change the x_1 to a larger number the time it takes to iterate is shorter from when x_1 is smaller. So if x_1 is smaller the iteration time becomes large although still causing chaos at $r = 3.57$. This differs from smaller values of R because chaos does not occur at the smaller R values.