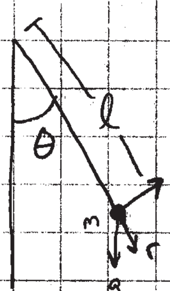
4-28-09

The Nonlinear Pendulum

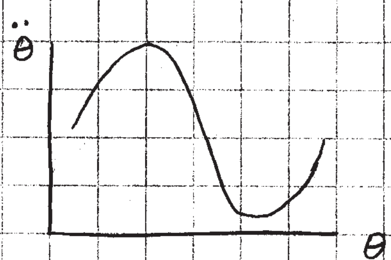
$$m\ell \frac{d^2\theta}{dt^2} = mg\sin\theta - c\ell\frac{d\theta}{dt}$$

θ and t are variables
m, ℓ, and g are parameters
c is the parameter for dampening



When there is no dampening, and when other parameters are equal to one:

$$\ddot{\theta} + \sin(\theta) = 0$$



- Angular acceleration as a function of theta

$$m\ell \frac{d^2\theta}{dt^2} = mg\sin\theta - c\ell\frac{d\theta}{dt}$$

Increasing gravity decreases the period of oscillation
Increasing the length of the pendulum increases the period of oscillation

For small values of θ:

$$\sin(\theta) \sim \theta \qquad \sin(\theta) = \left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} \dots\right)$$

$$\ddot{\theta} = -\frac{g}{\ell}\theta$$

$$\ddot{\theta} + \frac{g}{l}\theta = 0$$

$$\lambda^2 + \frac{g}{l} = 0 \rightarrow \lambda^2 = -\frac{g}{l} \rightarrow \lambda = \pm\sqrt{\frac{g}{l}}\, i$$

$$\theta = c_1 \cos\left(\sqrt{\frac{g}{l}}\,t\right) + c_2 \sin\left(\sqrt{\frac{g}{l}}\,t\right)$$

Period of oscillation

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Units of T

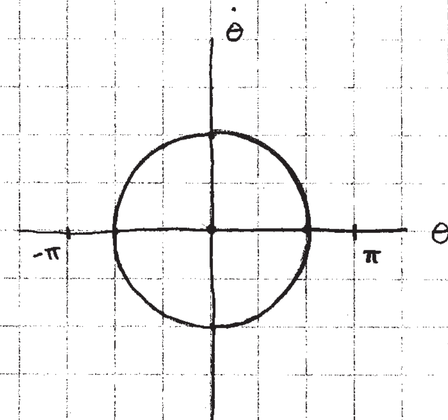
$$T = \underset{\substack{\uparrow \\ \text{constant} \\ \text{no units}}}{c}\sqrt{l/g} = \sqrt{\frac{m}{m/s^2}} = \sqrt{s^2} = s$$

Plotting $\dot{\theta}$ as a function of θ for a pendulum

$\frac{d\theta}{dt}$ = angular velocity = v

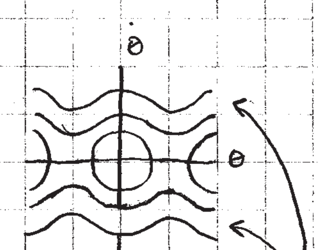
$\frac{dv}{dt}$ = angular acceleration = $-\frac{g}{l}\sin(\theta)$

$$\begin{cases} \frac{d\theta}{dt} = v \\ \frac{dv}{dt} = -\frac{g}{l}\sin(\theta) \end{cases}$$



At stable equilibrium,
$\theta = 0 \quad \dot{\theta} = 0$

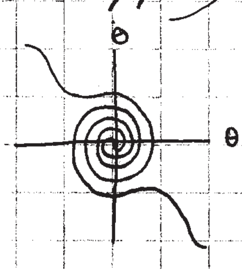
During Motion,
when $\theta = 0$, $\dot{\theta}$ = max
when θ = max, $\dot{\theta} = 0$



At unstable equilibrium
$\theta = \pi \quad \dot{\theta} = 0$

When the pendulum rotates continuously, $\dot{\theta}$ is never θ

continuous rotation



With dampening, the function resembles spirals