Fourier series

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos\left(n\frac{\pi x}{L}\right) + b_n \sin\left(n\frac{\pi x}{L}\right) \right],$$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\ dx, \quad a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(n\frac{\pi x}{L}\right)\ dx, \quad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(n\frac{\pi x}{L}\right)\ dx.$$

Fourier cosine series

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos\left(n\frac{\pi x}{L}\right) \right],$$

$$a_0 = \frac{1}{L}\int_{0}^{L} f(x)\ dx, \qquad a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\left(n\frac{\pi x}{L}\right)\ dx.$$

Fourier sine series

$$f(x) = \sum_{n=1}^{\infty} \left[b_n \sin\left(n\frac{\pi x}{L}\right) \right], \qquad b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\left(n\frac{\pi x}{L}\right)\ dx.$$

Complex form

$$f(x) = \sum_{n=-\infty}^{\infty} c_n \exp\left(i\, n\frac{\pi x}{L}\right), \qquad c_n = \frac{1}{2L}\int_{-L}^{L} f(x)\exp\left(-i\, n\frac{\pi x}{L}\right)\ dx.$$

Fourier transforms

$$\widehat{f}(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\exp(-ikx)\ dx, \qquad f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \widehat{f}(k)\exp(ikx)\ dk.$$

Convolution

$$(f * g)(x) = \int_{-\infty}^{\infty} f(x-t)g(t)\ dt = \int_{-\infty}^{\infty} f(t)g(x-t)\ dt.$$

$$\mathcal{F}(f * g) = \sqrt{2\pi}\ \mathcal{F}(f)\ \mathcal{F}(g).$$

Cosine transform

$$\widehat{f}(k) = \widehat{f}_c(k) = \sqrt{\frac{2}{\pi}}\int_{0}^{\infty} f(x)\cos(kx)\ dx, \qquad f(x) = \sqrt{\frac{2}{\pi}}\int_{0}^{\infty} \widehat{f}_c(k)\cos(kx)\ dk.$$

Sine transform

$$\widehat{f}_s(k) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin(kx)\ dx, \qquad f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty \widehat{f}_s(k) \sin(kx)\ dk.$$

Laplace transforms

$$\mathcal{L}(f)(s) = F(s) = \int_0^\infty \exp(-s\,t)\, f(t)\ dt.$$

s-shifting

$$\mathcal{L}\left(e^{a\,t} f(t)\right)(s) = F(s-a), \qquad e^{a\,t} f(t) = \mathcal{L}^{-1}\left(F(s-a)\right)(t).$$

Laplace transform of derivatives

$$\mathcal{L}\left(f'\right)(s) = s\,\mathcal{L}(f)(s) - f(0), \qquad \mathcal{L}\left(f''\right)(s) = s^2 \mathcal{L}(f)(s) - s\, f(0) - f'(0).$$

Laplace transform of antiderivatives

$$\mathcal{L}\left(\int_0^t f(\tau)\ d\tau\right)(s) = \frac{1}{s}\,\mathcal{L}(f)(s), \qquad \int_0^t f(\tau)\ d\tau = \mathcal{L}^{-1}\left(\frac{1}{s}\,\mathcal{L}(f)(s)\right)(t).$$

t-shifting

$$\mathcal{L}\left(f(t-a)\, H(t-a)\right)(s) = e^{-as} \mathcal{L}(f)(s), \qquad f(t-a)\, H(t-a) = \mathcal{L}^{-1}\left(e^{-as} \mathcal{L}(f)(s)\right)(t).$$

Differentiation of Laplace transforms

$$\mathcal{L}\left(t\, f(t)\right)(s) = -F'(s), \qquad \mathcal{L}^{-1}\left(F'(s)\right)(t) = -t\, f(t).$$

Integration of Laplace transforms

$$\mathcal{L}\left(\frac{f(t)}{t}\right)(s) = \int_s^\infty F(\nu)\ d\nu, \qquad \mathcal{L}^{-1}\left(\int_s^\infty F(\nu)\ d\nu\right)(t) = \frac{f(t)}{t}.$$