## Partial Differential Equations

Clicker questions
What is the order of the heat equation, $u_{t}=c^{2} u_{x x}$ ?

1. 1

マ2. 2
3. 3
4. 4

Is the two-dimensional wave equation, $\mathrm{u}_{\mathrm{tt}}=\mathrm{c}^{2}\left(\mathrm{u}_{\mathrm{xx}}+\mathrm{u}_{\mathrm{yy}}\right)$ linear?
$\checkmark 1$ Yes
2. No

Is Laplace's equation, $\mathrm{u}_{\mathrm{xx}}+\mathrm{u}_{\mathrm{yy}}=0$, homogeneous?
, 1. Yes
2. No


Is $f(x+c t)$, where $f$ is an arbitrary smooth function, $a$ solution of the one-dimensional wave equation

$$
u_{\mathrm{tt}}=\mathrm{c}^{2} \mathrm{u}_{\mathrm{xx}} ?
$$

, 1. Yes
2. No


Is $f(x+c t)+g(x-c t)$, where $f$ and $g$ are arbitrary smooth functions, a solution of the one-dimensional wave equation

$$
u_{\mathrm{tt}}=\mathrm{c}^{2} \mathrm{u}_{\mathrm{xx}} ?
$$

$\checkmark$ 1. Yes
2. No

1. Yes
2. No
$\mathrm{g}(\mathrm{x}-\mathrm{ct})$, where g is an arbitrary smooth function, a solution of the one-dimensional wave equation

$$
u_{t t}=c^{2} u_{x x} ?
$$



Consider the wave equation, $u_{\mathrm{tt}}=\mathrm{c}^{2} \mathrm{u}_{\mathrm{xx}}$, with Dirichlet boundary conditions on $[0,1]$, and initial condition given below. What is the coefficient of $\sin (3 \pi x) \cos (3 c \pi t)$ in the normal-mode expansion of the solution?
$u(x, 0)=\left\{\begin{array}{cc}\frac{x}{5} & \text { if } 0 \leq x \leq 0.5 \\ \frac{1-x}{5} & \text { if } 0.5 \leq x \leq 1\end{array}, \quad \frac{\partial u}{\partial t}(x, 0)=0\right.$

1. $4 /\left(5 \pi^{2}\right)$
2. $-4 /\left(45 \pi^{2}\right)$
3. $-4 /\left(9 \pi^{2}\right)$
4. 0


Consider the wave equation, $\mathrm{u}_{\mathrm{tt}}=\mathrm{c}^{2} \mathrm{u}_{\mathrm{xx}}$, with Dirichlet boundary conditions on $[0,1]$, and initial condition given below. What is the coefficient of $\sin (4 \pi x) \cos (4 \mathrm{c} \pi \mathrm{t})$ in the normal-mode expansion of the solution?
$u(x, 0)=\left\{\begin{array}{cc}\frac{x}{5} & \text { if } 0 \leq x \leq 0.5 \\ \frac{1-x}{5} & \text { if } 0.5 \leq x \leq 1\end{array}, \quad \frac{\partial u}{\partial t}(x, 0)=0\right.$

1. $4 /\left(5 \pi^{2}\right)$
2. $-4 /\left(45 \pi^{2}\right)$
3. $-4 /\left(9 \pi^{2}\right)$
4. 0


Consider the wave equation, $u_{t t}=c^{2} u_{x x}$, with Dirichlet boundary conditions on [0, 1], and initial condition given below. What is the coefficient of $\sin (5 \pi \mathrm{x}) \cos (5 \mathrm{c} \pi \mathrm{t})$ in the normal-mode expansion of the solution?
$u(x, 0)=\left\{\begin{array}{cc}\frac{x}{5} & \text { if } 0 \leq x \leq 0.5 \\ \frac{1-x}{5} & \text { if } 0.5 \leq x \leq 1\end{array}, \quad \frac{\partial u}{\partial t}(x, 0)=0\right.$

1. $4 /\left(125 \pi^{2}\right)$
2. $-4 /\left(45 \pi^{2}\right)$
3. $-4 /\left(9 \pi^{2}\right)$
4. 0


What is the dot product $\langle\mathrm{f}, \mathrm{f}\rangle=\|\mathrm{f}\|^{2}$, where $f(x, y)=\sin (m \pi x / a) \sin (n \pi y / b), m, n=1,2, \ldots ?$

How should k be chosen, if we want the solution F to $\mathrm{F}^{\prime \prime}=\mathrm{kF}$ to satisfy Dirichlet boundary conditions on [0, L]?

1. $\mathrm{k}=-\mathrm{n} \pi / \mathrm{L}$
2. $\mathrm{k}=-(\mathrm{n} \pi / \mathrm{L})^{2}$
3. $\mathrm{k}=-(\mathrm{n} \pi /(2 \mathrm{~L}))^{2}$


What is the solution to $\mathrm{dG} / \mathrm{dt}=\mathrm{kc}^{2} \mathrm{G}$ ?

1. $\mathrm{G}(\mathrm{t})=\mathrm{G}(0) \exp \left(-\mathrm{kc}^{2} \mathrm{t}\right)$
2. $G(t)=G(0) \exp \left(-c^{2} t\right)$

マ3. $G(t)=G(0) \exp \left(k^{2} t\right)$

How should k be chosen, if we want the solution F to $\mathrm{F}^{\prime \prime}=\mathrm{kF}$ to satisfy Neumann boundary conditions on [ $0, \mathrm{~L}]$ ?

1. $\mathrm{k}=-\mathrm{n} \pi / \mathrm{L}$
2. $\mathrm{k}=-(\mathrm{n} \pi / \mathrm{L})^{2}$
3. $\mathrm{k}=-(\mathrm{n} \pi /(2 \mathrm{~L}))^{2}$

