1. Consider the dynamical system

$$\frac{dx}{dt} = (r^2 + 4r + x^2 - 2x + 4)(x - 1)(x - r).$$

(a) What is the dimension of this dynamical system? Is it linear or nonlinear? Justify your answer.

(b) Plot the bifurcation diagram of this dynamical system, identify the bifurcations, as well as the corresponding critical values of r. Use the MATLAB-based application `Plotter` to check your answer.

(c) Find the normal form for each of the bifurcations you identified in part (b). Show all your work.

(d) Draw the phase portrait of this system for $r = -2$ and for $r = 2$. Use `PPLANE` to check your answer.

2. For each part below, find a two-dimensional dynamical system of the form

$$\frac{dX}{dt} = MX,$$

where the entries of the matrix M are real and all non-zero, and which satisfies the given property. Check your answer with `PPLANE`.

(a) The origin is a saddle point.

(b) The origin is a degenerate node, and M has an eigenvalue $\lambda = 1$ with multiplicity 2.

(c) The origin is a stable node.

3. Problems from the book: Plot the phase portrait and classify the fixed point of the following linear systems. If the eigenvectors are real, indicate them in your sketch. Check your answers with `PPLANE`.

(a) **5.2.3** $\dot{x} = y,\ \dot{y} = -2x - 3y.$

(b) **5.2.5** $\dot{x} = 3x - 4y,\ \dot{y} = x - y.$

(c) **5.2.6** $\dot{x} = -3x + 2y,\ \dot{y} = x - 2y.$

(d) **5.2.8** $\dot{x} = -3x + 4y,\ \dot{y} = -2x + 3y.$

4. Work through problems **5.2.1** and **5.2.2** from the book. Use `PPLANE` to plot the phase portraits and to check your answers.