1. Find the general solution of the following system of differential equations,

$$\frac{dX}{dt} = BX, \qquad \text{where } X(t) \in \mathbb{R}^3 \text{ and } \qquad B = \begin{bmatrix} -1 & -4 & -4 \\ 2 & 4 & 2 \\ 0 & 1 & 3 \end{bmatrix}.$$

2. Find the general solution of the following system of differential equations,

$$\frac{dX}{dt} = CX, \qquad \text{where } X(t) \in \mathbb{R}^3 \text{ and } \qquad C = \begin{bmatrix} 0 & -2 & -2 \\ 3 & 6 & 4 \\ -2 & -3 & -1 \end{bmatrix}.$$

3. Give three linearly independent solutions to the system of differential equations defined in question 1 above. Show that these solutions are indeed linearly independent.

4. Give three linearly independent solutions to the system of differential equations defined in question 2 above. Show that these solutions are indeed linearly independent.

5. In each case below, find a 2×2 matrix M **whose entries are all non-zero**, and which satisfies the given property.

(a) The eigenvalues of M are 3 and 5.

(b) The eigenvalues of M are -3 and 2.

(c) The matrix M has an eigenvalue $\lambda = -2$ of multiplicity 2, but the corresponding (genuine) eigenspace is one-dimensional.

6. Let M be the matrix you found in question 5.(c) above.

(a) Write down the general solution to

$$\frac{dX}{dt} = MX, \qquad X(t) \in \mathbb{R}^2.$$

(b) Would you say that the fixed point of this system, $X_0 = (0,0)^T$, is stable or unstable? Justify your answer.

7. For each part below, write down a system of differential equations of the form

$$\frac{dX}{dt} = MX, \qquad X(t) \in \mathbb{R}^2,$$

where the 2×2 matrix M has constant coefficients, and which satisfies the given property.

(a) All solutions with initial conditions different from $(0,0)^T$ move away from the origin $(0,0)^T$. Justify your answer.

(b) There is a non-trivial set of initial conditions such that the solution converge towards the origin, $(0,0)^T$, and there is a set of initial conditions such that the solution moves away from the origin. Justify your answer.