1. Find the determinant of the following matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}.$$

2. Find the eigenvalues and eigenvectors of the following matrix

$$B = \begin{bmatrix} -1 & -4 & -4 \\ 2 & 4 & 2 \\ 0 & 1 & 3 \end{bmatrix}.$$

3. Find the eigenvalues and (generalized) eigenspaces of the following matrix

$$C = \begin{bmatrix} 0 & -2 & -2 \\ 3 & 6 & 4 \\ -2 & -3 & -1 \end{bmatrix}.$$

4. Is the following set of vectors linearly independent? Justify your answer.

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \right\}$$

5. Consider the set of polynomials $\{p_1,\ p_2,\ p_3,\ p_4\}$, where

$$p_1(x) = 3; \qquad p_2(x) = 4x^2 - 6; \qquad p_3(x) = x^3 - 3x + 7; \qquad p_4(x) = x^4.$$

(a) Is this set linearly independent? Why or why not?

(b) Does this set span the space of polynomials of degree 5? Why or why not?

6. In each case below, find a 2×2 matrix M **whose entries are all non-zero**, and which satisfies the given property.

(a) The eigenvalues of M are 1 and 2.

(b) The eigenvalues of M are -1 and 1.

(c) The matrix M has an eigenvalue $\lambda = 3$ of multiplicity 2, but the corresponding eigenspace is one-dimensional.