Ground State Energy of the Discrete Random Schrödinger Operator with Bernoulli Potential

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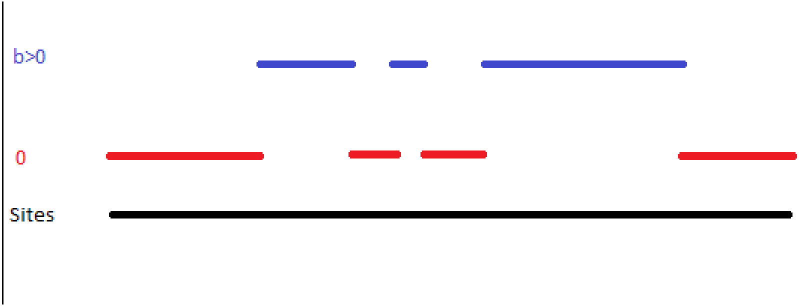
Consider a system on the one-dimensional lattice length L and discrete operator $H = -\Delta + V$ with Bernoulli potential:

$$V_i = \begin{cases} 0 & : p \\ b & : q = 1 - p \end{cases}$$

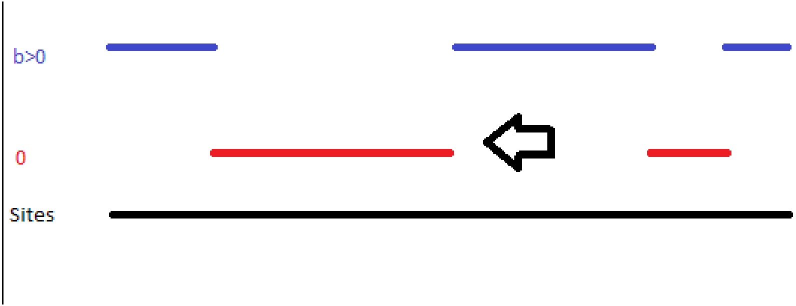


We are interested in the **Ground State Energy** E_0 and corresponding wave function ψ_0 in $\ell^2(\{1, \ldots, L\})$.

$$E_0 = \min_{\{\|\phi\|=1\}} \langle \phi, H\phi \rangle = \min_{\{\|\phi\|=1\}} \sum_x |\phi'(x)|^2 + |V\phi(x)|^2$$

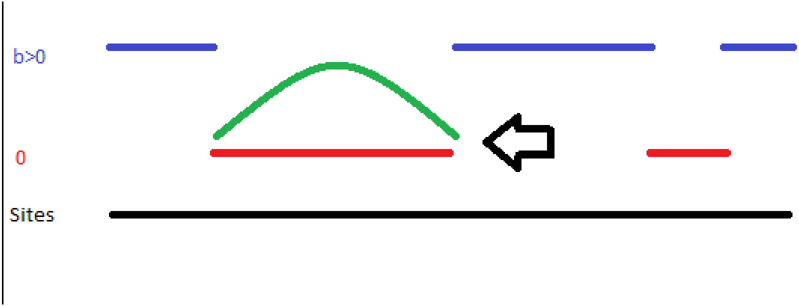
What determines the energy and the associated wave function?

The longest ‘island’ of zero potential in the system is the important geometric feature.



This island is not necessarily unique. We denote the length of this island ℓ_L.

As a test function, we place a half sine wave on this island:



The energy of this test function is bounded by $\frac{\pi^2}{(\ell_L+1)^2}$. This bounds the ground state energy.

This energy bound gives a bound on the norm of the ground state on b potential.

$$\|\psi_0|_b\|^2 \leq \frac{\pi^2}{b(\ell_L + 1)^2}$$

In the large system limit, the quantity ℓ_L goes to infinity with probability one.

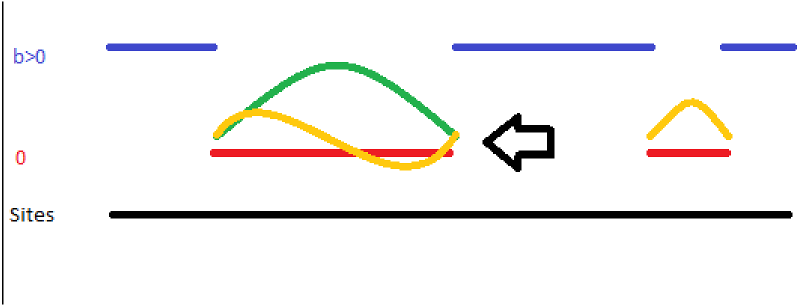
With some careful technical work, the lower bound can be found as

$$\left(1-\frac{3\pi^2}{b\ell_N^\gamma}\right)\left(1-\frac{1}{\ell_N^{(1-\gamma)/2}}\right)^2\left(\frac{\pi^2}{(\ell_N+1)^2}+O\left(\frac{\pi^4}{(\ell_N+1)^4}\right)\right)\leq E_0^N$$

where $\gamma\in(0,1)$ is a parameter that can be chosen.

$$P\left[\lim_{L\to\infty}\frac{E_0}{\frac{\pi^2}{(\ell_L+1)^2}}\right]=1$$

Conjecture: For excited states, we expect similar inequalities to arise.



Intuition: Finite barriers appear infinite to low energy states in the large system limit.

This may give a strong Lifschitz Tail. (V. Borovyk)

This method generalizes to interacting systems.

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