Data assimilation and inverse problems – Homework Set 8

Particle filters

1. Consider a stochastic version of the Lorenz’63 model given by:

\[ x_k = f(x_{k-1}) + \sqrt{\Delta t} \, v_k, \quad v_k \sim \mathcal{N}(0, I), \text{ iid}, \]

where

\[ f(x) = x_k + \Delta t \begin{pmatrix} \sigma(x_k^2 - x_k^1) \\ x_k^1(p - x_k^3 - x_k^2) \\ x_k^1x_k^2 - \beta x_k^3 \end{pmatrix} \]

where \( x_k = (x_k^1, x_k^2, x_k^3)^T \), \( \sigma = 10 \), \( \beta = 8/3 \), \( p = 28 \). Use a time step of \( \Delta t = 0.01 \).

(a) You have observations of \( x^1 \) and \( x^3 \) every \( \Delta T = 0.01 \) time units. The observation noise is Gaussian with zero mean and covariance \( R = I \). Implement a “standard” particle filter, an optimal particle filter, and an EnKF. How do these three methods compare?

(b) You have observations of \( x^1 \) and \( x^3 \) every \( \Delta T = 0.1 \) time units. The observation noise is Gaussian with zero mean and covariance \( R = I \). Implement a “standard” particle filter and an EnKF. How do these three methods compare?

2. Try a particle filter on the Lorenz’95 model (see HW 3). Does it “work”? How many particles do you need?