Math 128b – Spring 2014 – Homework set 9
Due Tuesday 4/8 in class before the lecture starts.

Problem 1
(a) Code our workhorse algorithm for computing eigenvalues and eigenvectors of a square matrix. First bring $A$ into upper Hessenberg form (using Householder reflectors, you need to write your own code for this). Then apply shifted QR with inflation to find all eigenvalues of $A$ (you can use Matlab’s function “qr” for the required QR factorizations). Then apply one step of inverse power iteration to find the eigenvectors of $A$.

(b) Write code to compute the SVD of $A$. First compute eigenvalues and eigenvectors of

\[ B = \begin{pmatrix} 0 & A^T \\ A & 0 \end{pmatrix}, \]

using your code from (a). Then use the results (eigenvalues and eigenvectors of $B$) to construct the SVD of the matrix $A$.

Test your codes on the matrix

\[ A = \begin{pmatrix} 7 & -33 & -15 \\ 2 & 26 & 7 \\ -4 & -50 & -13 \end{pmatrix}, \]

and compare with Matlab’s commands “eig(A)” and “svd(A)” give you. You should hand in your codes for the workhorse QR algorithm (including the step with putting $A$ into upper Hessenberg form) and for your SVD, as well as the results of these codes when applied to $A$ above.

Problem 2
Let $A$ be an $n \times n$ symmetric matrix.

(a) Show that the eigenvalues of $A$ are real.

(b) Show that the eigenvectors of $A$ can be chosen real.

(c) Show that if all eigenvalues are distinct, then the eigenvectors of $A$ are orthogonal (the assumption of distinct eigenvalues makes the proof easy, but the statement is true even if some eigenvalues are repeated).

(d) Show that $A$ in upper Hessenberg form is tridiagonal.

(e) Express the SVD of $A$ in terms of the eigenvalues and eigenvectors of $A$. In particular show that the singular values of $A$ are the absolute values of the eigenvalues of $A$.

Problem 3
Let $A$ be a real $m \times n$ matrix.

(a) Show that $A^T A$ and $AA^T$ have the same nonzero eigenvalues.

(b) Show that the nonzero singular values of $A$ are the square roots of the eigenvalues of $A^T A$ or $AA^T$. 