MATH 125: NEW FUNCTIONS FROM OLD FUNCTIONS

FALL 2009

Understanding more *general functions* by relating them to certain *basic functions* is an essential skill one should develop in calculus. Once a basic graph is understood, more complicated looking *translations* , *reflections*, and *stretches* are also easily understood.

Translations: Suppose f is a function whose graph is known and let $c > 0$ is a constant.

(1) Vertical Shifts:
i) $y = f(x) + c$ shifts the graph of $y = f(x)$ a distance c units upwards.
ii) $y = f(x) - c$ shifts the graph of $y = f(x)$ a distance c units downwards.
Note: Vertical translations (or shifts) do not effect vertical asymptotes, but they will move horizontal asymptotes!

(2) Horizontal Shifts:
i) $y = f(x - c)$ shifts the graph of $y = f(x)$ a distance c units to the right.
ii) $y = f(x + c)$ shifts the graph of $y = f(x)$ a distance c units to the left.
Note: Horizontal translations (or shifts) do not effect horizontal asymptotes, but they will move vertical asymptotes!

Reflections: Suppose f is a function whose graph is known.
i) $y = -f(x)$ reflects the graph of $y = f(x)$ about the x-axis. This kind of reflection will not effect vertical asymptotes, but it may effect horizontal asymptotes.
ii) $y = f(-x)$ reflects the graph of $y = f(x)$ about the y-axis. This kind of reflection will not effect horizontal asymptotes, but it may effect vertical asymptotes.

Stretches: Suppose f is a function whose graph is known and let $c > 0$ be a constant.

(1) Vertical Stretching:
i) If $c > 1$, then the graph of $y = cf(x)$ will be stretched vertically; the stretching is proportional to c units.
ii) If $0 < c < 1$, then the graph of $y = cf(x)$ will be shrunk vertically; the shrinking is proportional to c units.
Note: Vertical stretching will effect horizontal asymptotes, but not vertical asymptotes.

(2) Horizontal Stretching:
i) If $c > 1$, then the graph of $y = f(cx)$ will be shrunk horizontally; the "old" function values are achieved "faster", at a rate that is proportional to c units.
ii) If $0 < c < 1$, then the graph of $y = f(cx)$ will be stretched horizontally; the "old" function values are achieved more slowly, at a rate that is proportional to c units.
Note: Horizontal stretching will effect vertical asymptotes, but not horizontal asymptotes.

Warning: It is very important to interpret *combinations* of translations, reflections, and stretches carefully. In general, given a function f, the graph of f shifted first and then reflected is *different* from the graph of f reflected first and then shifted!! Here is an example:

Example: Let $y = f(x) = e^x$. We will calculate two *different* functions.

i) **In Words:** Take f, shift by three units to the right, and then reflect the result about the y-axis.

In Formulas: Let $g(x) = f(x - 3) = e^{x-3}$, and then $h(x) = g(-x) = e^{-x-3}$.

Compare this with the following function.

ii) **In Words:** Take f, reflect about the y-axis, and then shift the result three units to the right.

In Formulas: Let $g(x) = f(-x) = e^{-x}$, and then $h(x) = g(x - 3) = e^{-(x-3)} = e^{-x+3}$.

These two functions are different!

Exercise 1. *Let the function f be given by $f(x) = x^2$.*

Graph the following translations:

i) $y = f(x) + 3 = x^2 + 3$,
ii) $y = f(x) - 4 = x^2 - 4$,
iii) $y = f(x - 2) = (x - 2)^2$,
iv) $y = f(x + 1) = (x + 1)^2$.

Graph the following reflections:

i) $y = -f(x) = -x^2$,
ii) $y = f(-x) = (-x)^2$,

Graph the following combinations:

i) $y = -f(x + 3) + 2 = -(x + 3)^2 + 2$,
ii) $y = f(-(x - 3)) - 1 = (-x + 3)^2 - 1$.

Exercise 2. *Let the function g be given by $g(x) = 3^x$.*

Graph the following translations:

i) $y = g(x) + 3 = 3^x + 3$,
ii) $y = g(x) - 4 = 3^x - 4$,
iii) $y = g(x - 2) = 3^{x-2}$,
iv) $y = g(x + 1) = 3^{x+1}$.

Graph the following reflections:

i) $y = -g(x) = -3^x$,
ii) $y = g(-x) = 3^{-x}$,

Graph the following combinations:

i) $y = -g(x + 3) + 2 = -3^{x+3} + 2$,
ii) $y = g(-(x - 3)) - 1 = 3^{-x+3} - 1$.