MATH 129:
TEST 2 MAKE-UP

SPRING 2019

<table>
<thead>
<tr>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Key</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>I.D. Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Key</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Total</td>
<td>60</td>
<td></td>
</tr>
</tbody>
</table>
Directions: This work is an optional assignment for those who took the second test on Tuesday, March 19, 2019. It is due on Thursday, March 28, 2019 before the beginning of class. **No late work will be accepted; one minute late to class is late.** If you turn this in, I will grade it (with a score out of 100) and your new grade on test 2 will be the average of the two scores you have received. If you do not turn this in, your grade on test 2 will stay the same.

Show all work on calculating the integrals below, unless you are told you can use the integration table. When you use the integration table, indicate which number you are using.

(1) Determine if the following integral converges or diverges. If it converges, find its value.

\[ \int_{-\infty}^{-4} e^{2x+3} \, dx \]

This is a type I improper integral:

\[ \int_{-\infty}^{-4} e^{2x+3} \, dx = \lim_{a \to -\infty} \int_{a}^{-4} e^{2x+3} \, dx \]

\[ u = 2x+3 \]

\[ du = 2 \, dx \]

\[ \int_{a}^{-4} e^{2x+3} \, dx = \frac{1}{2} \int_{a}^{-4} e^{u} \, du \]

\[ e^{2x+3} \]

\[ \int_{a}^{-4} e^{u} \, du = \frac{1}{2} (e^{u} \bigg|_{a}^{-4}) \]

\[ = \frac{1}{2} (e^{-5} - e^{2a+3}) \]

\[ = \frac{e^{-5}}{2} \]

The integral converges to this value.
MATH 129:
TEST 2 MAKE-UP

SPRING 2019

<table>
<thead>
<tr>
<th>Name</th>
<th>Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.D. Number</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>60</td>
<td></td>
</tr>
</tbody>
</table>
Directions: This work is an optional assignment for those who took the second test on Tuesday, March 19, 2019. It is due on Thursday, March 28, 2019 before the beginning of class. **No late work will be accepted; one minute late to class is late.** If you turn this in, I will grade it (with a score out of 100) and your new grade on test 2 will be the average of the two scores you have received. If you do not turn this in, your grade on test 2 will stay the same.

Show all work on calculating the integrals below, unless you are told you can use the integration table. When you use the integration table, indicate which number you are using.

1. Determine if the following integral converges or diverges. If it converges, find its value.

\[ \int_{-\infty}^{-4} e^{2x+3} \, dx \]

This is a type I improper integral:

\[ \int_{-\infty}^{-4} e^{2x+3} \, dx = \lim_{a \to -\infty} \int_{a}^{-4} e^{2x+3} \, dx \]

Let \( u = 2x + 3 \)

\[ du = 2 \, dx \]

\[ \int_{a}^{-4} e^{2x+3} \, dx = \lim_{a \to -\infty} \int_{u(a)}^{-5} e^u \, \frac{du}{2} \]

\[ = \lim_{a \to -\infty} \frac{1}{2} \left( e^{-5} - e^{-2a+3} \right) \]

\[ = \frac{e^{-5}}{2} \]

The integral converges to this value.
(2) Consider the following integral.

\[ \int_0^1 \frac{3x + 5}{\sqrt{1 + x^2}} \, dx. \]

This is a Type II improper integral due to a vertical asymptote at \( x = 0 \).

a) Predict whether or not this integral converges or diverges.

\[ \frac{3x + 5}{\sqrt{x} + x^3} \approx \frac{3x + 5}{x \sqrt{1 + x^2}} \quad \text{near } x = 0 \]

For all \( 0 < x \leq 1 \),

\[ \sqrt{x} \leq \sqrt{x} + x^3 \implies \frac{1}{\sqrt{x} + x^3} \leq 1 \]

Since \( 3x + 5 \geq 0 \), we have that

\[ \frac{3x + 5}{\sqrt{x} + x^3} \leq \frac{3x + 5}{\sqrt{x}} = 3\sqrt{x} + \frac{5}{\sqrt{x}} = g(x) \]

By direct calculation (and the p-test),

\[ \int_0^1 g(x) \, dx \text{ converges} \]

So

\[ \int_0^1 \frac{3x + 5}{\sqrt{x} + x^3} \, dx \text{ converges by comparison.} \]
(3) Consider a solid whose base is the region bounded by the curves \( y = -x^2 + 3 \) and \( y = 2x - 5 \), with cross-sections perpendicular to the \( y \)-axis that are squares.

   a) Sketch the base of this solid.

   \[
   -x^2 + 3 = 2x - 5 \\
   0 = x^2 + 2x - 8 \\
   = (x+4)(x-2) \\
   x = -4, 2
   \]

   

   b) Find a Riemann sum which approximates the volume of this solid.

   Riemann Sum for the volume

   Note:
   \( y = 2x - 5 \Rightarrow 2x = y + 5 \Rightarrow x = \frac{y+5}{2} \)

   \[
   y = -x^2 + 3 \\
   \Rightarrow x^2 = 3 - y \\
   \Rightarrow x = \pm \sqrt{3-y} \\
   \]

   \[
   \text{Volume} = \int_{-1}^{3} \left( \frac{y+5}{2} + \sqrt{3-y} \right)^2 \, dy \\
   = \int_{-1}^{3} \left( \frac{y+5}{2} \right)^2 \, dy + \int_{-1}^{3} \left( \frac{\sqrt{3-y}}{2} \right)^2 \, dy
   \]

   c) Write a definite integral that calculates this volume precisely. You do not have to calculate the integral.
(4) Sketch the circle of radius 1 centered at the origin and the circle of radius 1 centered at the point \((1,0)\) both on the same axis.

\[ x^2 + y^2 = 1 \quad \text{and} \quad (x-1)^2 + y^2 = 1 \]

\[ \Rightarrow y = \pm \sqrt{1-x^2} \quad \Rightarrow \quad y = \pm \sqrt{1-(x-1)^2} \]

Where do they meet?

\[ \pm \sqrt{1-x^2} = \pm \sqrt{1-(x-1)^2} \quad \Rightarrow \quad 1-x^2 = 1-(x-1)^2 \]

\[ 1-x^2 = 1-x^2 + 2x + 1 \]

\[ \Rightarrow 0 = 2x - 1 \]

\[ \Rightarrow x = \frac{1}{2} \]

a) Write an integral which represents the area of the intersection of these circles (with \(x\)-axis integration). **You need not evaluate the integral.**

\[ \text{Area} = \int_{0}^{\frac{1}{2}} \left( \sqrt{1-(x-1)^2} - (-\sqrt{1-(x-1)^2}) \right) \, dx \]

\[ + \int_{\frac{1}{2}}^{1} \left( \sqrt{1-x^2} - (-\sqrt{1-x^2}) \right) \, dx \]

b) Write an integral which represents the area of the intersection of these circles (with \(y\)-axis integration). **You need not evaluate the integral.**

\[ \text{Area} = \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \left( \sqrt{1-y^2} - (1-\sqrt{1-y^2}) \right) \, dy \]

\[ + \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{\sqrt{3}}{2} \, dy \]

c) Write an integral which represents the volume of the solid obtained by revolving this region of intersection about the \(y\)-axis. **You need not evaluate the integral.**

\[ \text{Volume} = \pi \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} (\sqrt{1-y^2})^2 \, dy - \pi \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \left(1-\sqrt{1-y^2}\right)^2 \, dy \]

\[ + \pi \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \left(\sqrt{1-(x-1)^2}\right)^2 \, dx \]

d) Consider the region in the first quadrant inside the circle centered at the origin and outside the circle centered at \((1,0)\). Write an integral which represents the volume of the solid obtained by revolving this region about the \(x\)-axis. **You need not evaluate the integral.**

\[ \text{Volume} = \pi \int_{0}^{\frac{\sqrt{2}}{2}} (\sqrt{1-x^2})^2 \, dx \]

\[ - \pi \int_{0}^{\frac{1}{2}} \left(\sqrt{1-(x-1)^2}\right)^2 \, dx \]
(5) a) Write an integral for the arc length of the curve \( y = \frac{2}{5}\sqrt{25-x^2} \) from \( x = 0 \) to \( x = 4 \). **You need not evaluate the integral.** Approximate your answer with \( \text{LEFT}(2) \) and \( \text{RIGHT}(2) \).

\[
\begin{align*}
\frac{f(x)}{5} &= \frac{2}{5}\sqrt{25-x^2} \\
&= \frac{2}{5}(25-x^2)^{1/2} \\
f'(x) &= \frac{2}{5}\frac{1}{2}(25-x^2)^{-1/2}(-2x) \\
&= \frac{-2x}{5\sqrt{25-x^2}} \\
\Rightarrow f'(x)^2 &= \frac{4x^2}{25(25-x^2)} \\
\text{Arclength} &= \int_0^4 \sqrt{1 + \frac{4x^2}{25(25-x^2)}} \, dx \\
\text{Left}(2) &= \sqrt{1 + \left(\frac{f'(2)}{2}\right)^2} \cdot 2 + \sqrt{1 + \left(\frac{f'(0)}{2}\right)^2} \cdot 2 \\
&\approx 4.03 \\
\text{Right}(2) &= \sqrt{1 + \left(\frac{f'(2)}{2}\right)^2} \cdot 2 + \sqrt{1 + \left(\frac{f'(0)}{2}\right)^2} \cdot 2 \\
&\approx 4.30
\end{align*}
\]

b) Find an exact value for the arc length of the parametrized curve: \( x = \cos(t) \) and \( y = \sin(t) \) when \( 0 \leq t \leq 1 \).

\[
\begin{align*}
\text{Arclength} &= \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt \\
&= \int_0^1 \sqrt{(-\sin(t) \cdot e^t)^2 + (\cos(t) \cdot e^t)^2} \, dt \\
&= \int_0^1 e^t \sqrt{\sin^2(t) + \cos^2(t)} \, dt \\
&= \int_0^1 e^t \, dt = \left[ e^t \right]_0^1 \\
&= e - 1
\end{align*}
\]
(6) A water tank is in the shape of a cone with height 40 ft and base radius 8 ft. The cone rests on its base. Recall that 1 cubic foot of water weighs 62.4 pounds. For the problems below, you do not need to calculate the integrals. To receive any partial credit, you must show the work on the Riemann sum you use to determine the integral.

a) If the tank is full, set up an integral which calculates the work required to pump the water to the top (the height of the tip) of the tank.

\[
\text{Total work done} = \sum \text{work done on each slice} = \sum \left( \text{Force} \right) \left( \text{distance} \right)
\]

\[
= \sum \left( \text{density} \right) \left( \text{volume} \right) \left( \text{distance} \right)
\]

\[
= \sum \left[ \left( \frac{62.4}{\text{ft}^3} \right) \left( \frac{\pi r^2 \Delta h}{\text{ft}} \right) \right] \left( 40 - h \right)
\]

\[
\rightarrow \quad \int_{0}^{40} \frac{62.4 \cdot \pi}{8^3} (40 - h)^3 \, dh
\]

b) If the tank is filled to half its height, set up an integral which calculates the work required to pump the water 5 ft above the top of the tank.

\[
\text{Total work done} = \sum \text{work done on each slice} = \sum \left( \text{Force} \right) \left( \text{distance} \right)
\]

\[
= \sum \left( \text{density} \right) \left( \text{volume} \right) \left( \text{distance} \right)
\]

\[
= \sum \left[ \left( \frac{62.4}{\text{ft}^3} \right) \left( \frac{\pi r^2 \Delta h}{\text{ft}} \right) \right] \left( 45 - h \right)
\]

\[
\rightarrow \quad \int_{0}^{20} \frac{62.4 \cdot \pi}{8^3} (40 - h)^3 (45 - h) \, dh
\]