HW 1   Math 563
Due in class Wednesday, September 4, 2019.

1. Durrett (1.1.2). Let $\Omega = R, \mathcal{F} =$ all subsets so that A or A^c is countable. Suppose that $P(A) = 0$ in the first case, and $= 1$ in the second. Show that $(\Omega, \mathcal{F}, P)$ is a probability space.

2. Durrett (1.2.1) Suppose X and Y are random variables on $(\Omega, \mathcal{F}, P)$ and let $A \in \mathcal{F}$. Show that if we let $Z(\omega) = X(\omega)$ for $\omega \in A$ and $Z(\omega) = Y(\omega)$ for $\omega \in A^c$, then Z is a random variable.

3. Durrett (1.2.7) [Read (1.2.5) for a similar problem]. (i) Suppose X has a density function f. Compute the distribution function of X^2 and then differentiate to find its density function. (ii) Work out the answer when X has a standard Normal distribution to find the density of the chi-square distribution.

4. Durrett (1.6.3) (i) Show that Chebychev's inequality is sharp by showing that if $0 < b \leq a$ are fixed, there is an X with $EX^2 = b^2$ for which $P(|X| \geq a) = b^2/a^2$. (ii) Show that Chebychev's inequality is not sharp by showing that if X has $0 < EX^2 < \infty$, then

$$\lim_{a \to \infty} a^2 P(|X| \geq a)/EX^2 \; = \; 0.$$

5. Durrett (1.6.6). Let $Y \geq 0$ with $EY^2 < \infty$ and let $a < EY$. Apply the Cauchy-Schwarz inequality to $Y1_{(Y>a)}$ and conclude

$$P(Y > a) \; \geq \; (EY - a)^2/EY^2.$$

This is a useful lowerbound, often applied with $a = 0$.