HW 2 Math 563 Due Friday, Sep 20, 2019

1. (Durrett 1.6.14) Let $X \geq 0$, but do NOT assume that $E[1/X] < \infty$. Show

$$\lim_{y\to\infty} yE[1/X; X > y] = 0 \text{ and } \lim_{y\to 0} yE[1/X; X > y] = 0.$$

2. (Durrett 2.1.3) Recall, two events $A$ and $B$ are independent when $P(A \cap B) = P(A)P(B)$. Show that if $A, B$ independent then so are $A^c$ and $B$, and also $A^c$ and $B^c$.

3. (Durrett 2.1.4; see also problem 2.1.5 with respect to discrete r.v.'s) Suppose that $(X_1, \dots, X_n)$ has density $f(x_1, \dots, x_n)$, that is

$$P((X_1, \dots, X_n) \in A) = \int_A f(x)dx \text{ for } A \in \mathbb{B}_{\mathbb{R}^n}.$$

If $f$ can be written as $g_1(x_1) \dots g_n(x_n)$ where $g_i \geq 0$ measureable, then $X_1, \dots, X_n$ are independent. Note, it's not assumed that the $g_i$ are probability densities.

4. Consider the uniform probability space, $([0,1], \mathcal{B}_{[0,1]}, \lambda)$. Let $X(\omega) = \omega$ for $\omega \in [0,1]$, and define $Y = X(1 - X)$. Construct a non-constant r.v. $Z$ such that $Z$ and $Y$ are independent.