HW 4 Math/Stat 563 Due Wednesday Oct. 16, 2019

1. Let f be a bounded measurable function on $[0,1]$ that is continuous at $1/2$. Evaluate

$$\lim_{n\to\infty} \int_0^1 \int_0^1 \cdots \int_0^1 f\Big(\frac{x_1+x_2+\cdots+x_n}{n}\Big) dx_1 \cdots dx_n.$$

2. (Durrett 2.5.7) Let $\{X_n\}$ be independent r.v.'s. Suppose $\sum_{n=1}^{\infty} E|X_n|^{p(n)} < \infty$ where $0 < p(n) \leq 2$ for all $n \geq 1$, and $EX_n = 0$ when $p(n) > 1$. Show that $\sum_{n=1}^{\infty} X_n$ converges a.s.

3. Let $\{X_n\}$ be i.i.d with common distribution $F(x) = 1 - x^{-\alpha}$ for $x \geq 1$ and $\alpha > 0$. Let $M_n = \max_{m \leq n} X_n$. Then show that M_n/n^{γ} converges weakly as $n \uparrow \infty$ for some parameter γ. Identify γ and the limit distribution.