1. (Durrett 3.3.12 modified) Let $c$ be a constant. Show that $X_n \rightarrow c$ in probability if and only if $X_n \Rightarrow c$.

2. Let $\{X_n\}$ be iid and uniformly distributed on the finite set $\{1, 2, \ldots, m\}$. In repeated sampling, let $\nu_m$ be the time of the first coincidence, that is the time when an outcome is first repeated:

$$\nu_m = \inf \{n \geq 2 : X_n \in \{X_1, \ldots, X_{n-1}\}\}.$$

Show the weak convergence

$$\frac{\nu_m}{\sqrt{m}} \Rightarrow \nu$$

and compute $P(\nu > x)$ for $x > 0$.

3. (Durrett 3.3.1) Show that if $\varphi$ is a ch. f., then $\text{Re}(\varphi)$ and $|\varphi|^2$ are also ch. f.’s.