HW 6 Math/Stat 563 Due Wednesday, Nov. 13, 2019

1. (Durrett 3.4.1) Suppose you roll a die 180 times. Use Normal approximation (CLT) to estimate the probability that you will get fewer than 25 sixes.

2. (Durrett 3.4.4) Let $\{X_i\}$ be iid with $X_i \geq 0$, $E[X_i] = 1$ and $\text{Var}(X_i) = \sigma^2 \in (0, \infty)$. Show that $2(\sqrt{S_n} - \sqrt{n}) \Rightarrow \text{N}(0, \sigma^2)$. Here, $S_n = \sum_{i=1}^n X_i$ as usual.

3. (Durrett Section 3.9; 3.9.7) Recall Section 3.9 on weak convergence on $\mathbb{R}^d$/

Suppose $(X_1, \ldots, X_d)$ has a multivariate Normal distribution with mean vector θ and covariance Γ. Show that $X_1, \ldots, X_d$ are independent if and only if $\Gamma_{ij} = 0$ for $i \neq j$. In words, uncorrelated r.v.s *with a joint Normal distribution* are independent. [This is of course not the case with more general joint distributions.]

4. (Prisoner's Paradox) Three prisoners A,B, and C are on death row, each to be executed. But, then the governer decides to pardon one of the three and chooses at random the prisoner to pardon, and tells the warden. The next day, A asks the warden who has been pardoned– the warden refuses to tell. A then asks which of B or C will be executed (is not pardoned). The warden then says B will be executed.

A reasons that since B will be executed, then either A or C will be pardoned, and hence A's chance of being pardoned has risen to 1/2.

Do you agree or disagree? Explain with a proof.