Dynamical Structures in Iterative Decoding

Misha Stepanov

stepanov@math.arizona.edu

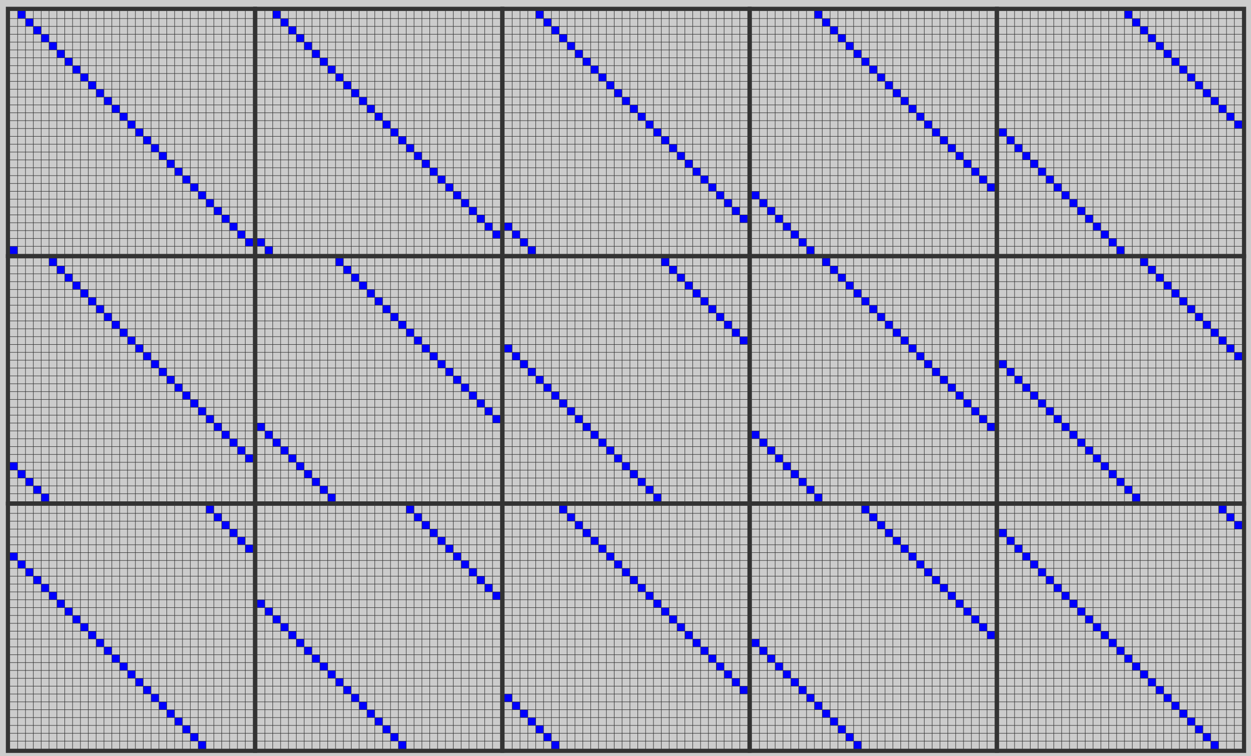
Department of Mathematics,
University of Arizona,
Tucson, AZ 85721, USA

2007-05-04 @ Algorithms, Inference, & Statistical Physics

Tanner's $[155, 64, 20]$ code

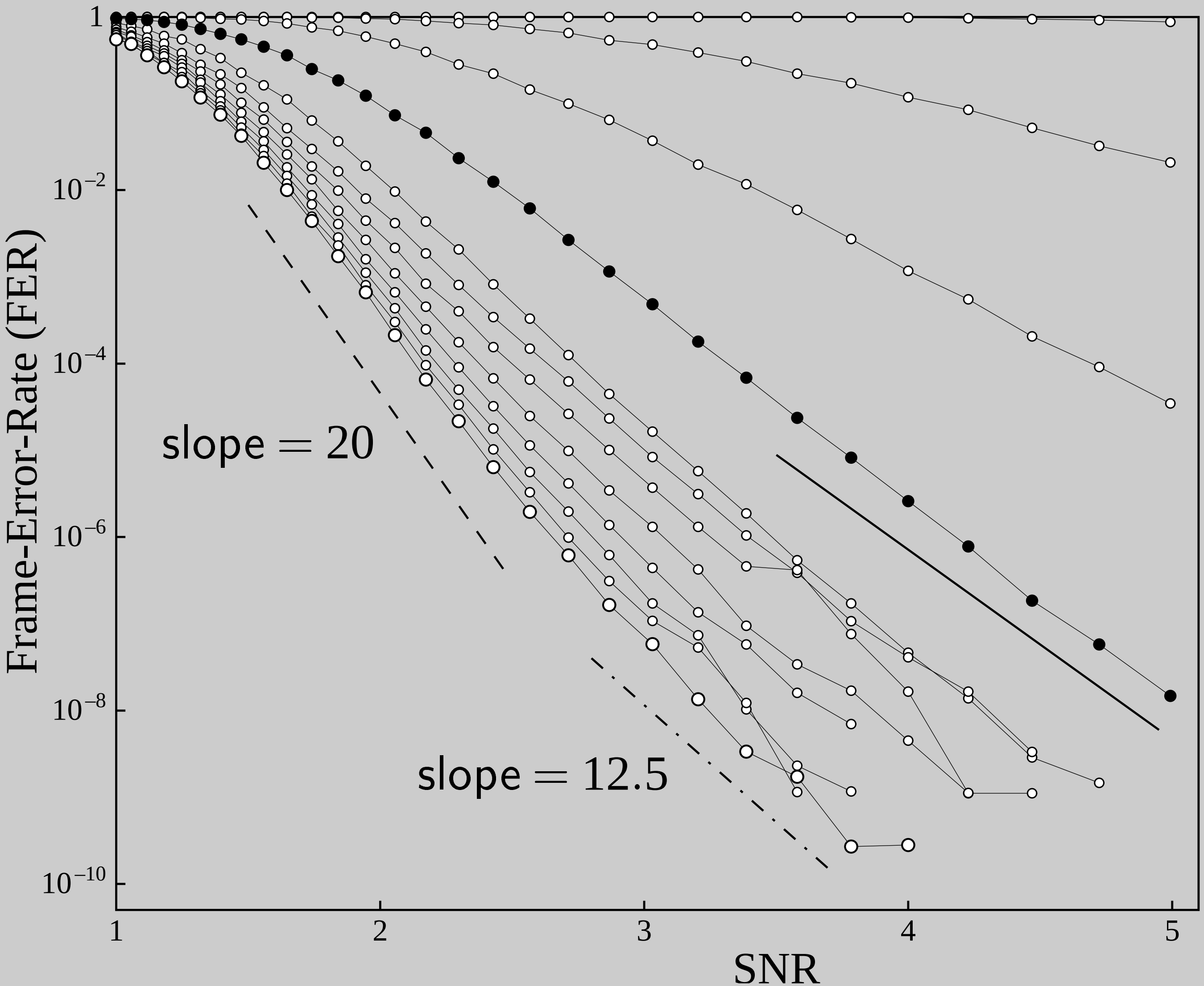
- 155: length of encoded message
- 64: informational bits
- 20: Hamming distance

Parity check matrix:



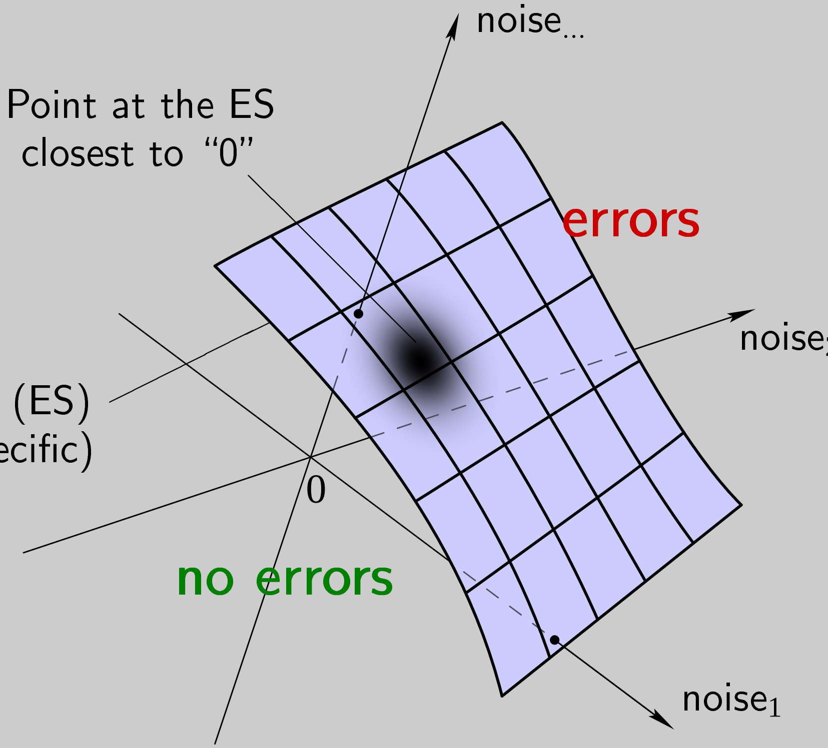
R.M. Tanner, D. Sridhara, T. Fuja, in *Proc. ISCTA 2001*
(Ambleside, UK, July 15–20, 2001), p. 365.

Frame-Error-Rate



Instanton method

instanton method
saddle-point method
Laplace method
method of steepest descent
large deviations



$$\mathrm{BER} = \int \mathrm{d(noise)}\, \mathrm{WEIGHT(noise)}$$

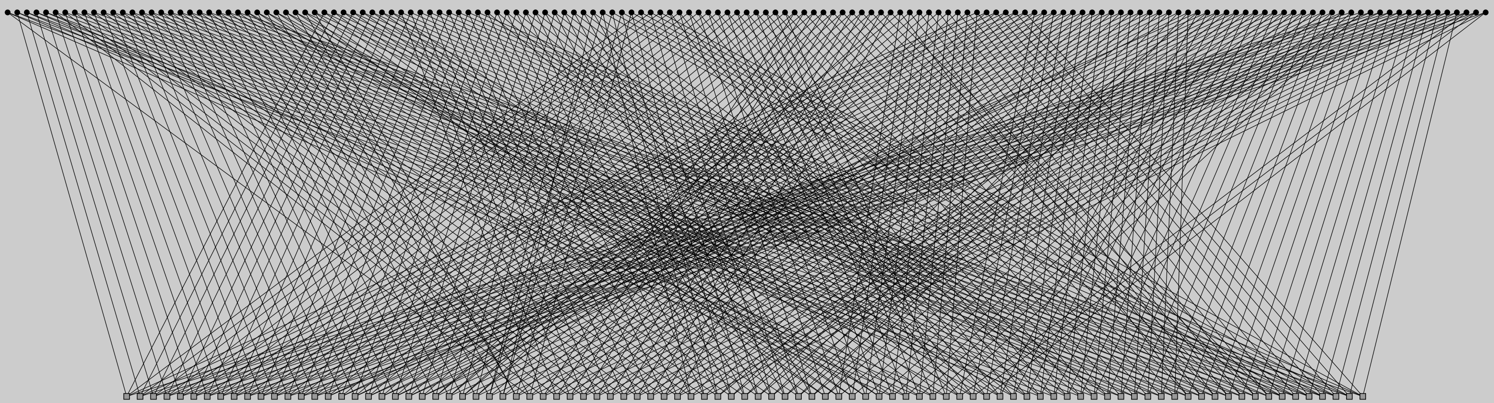
$$\mathrm{BER} \sim \mathrm{WEIGHT}\begin{pmatrix} \text{optimal conf} \\ \text{of the noise} \end{pmatrix}$$

$$\begin{matrix} \text{optimal conf} \\ \text{of the noise} \end{matrix} = \begin{matrix} \text{Point at the ES} \\ \text{closest to ``0''} \end{matrix}$$

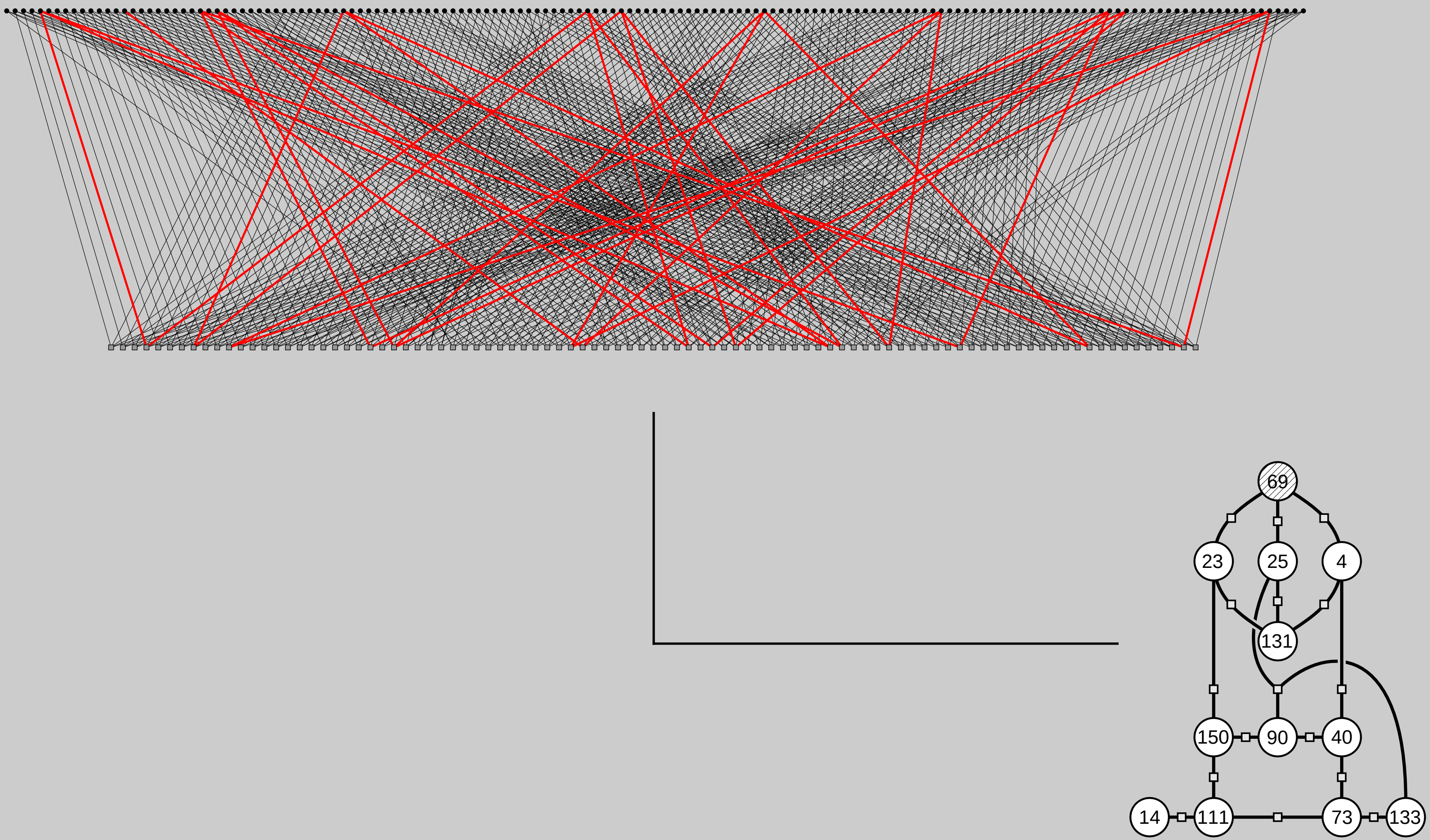
Chernyak, Chertkov, Stepanov, Vasic, Phys. Rev. Lett. **93**, 198702 (2004)

Stepanov, Chertkov, Chernyak, Vasic, Phys. Rev. Lett. **95**, 228701 (2005) [cond-mat/0506037]

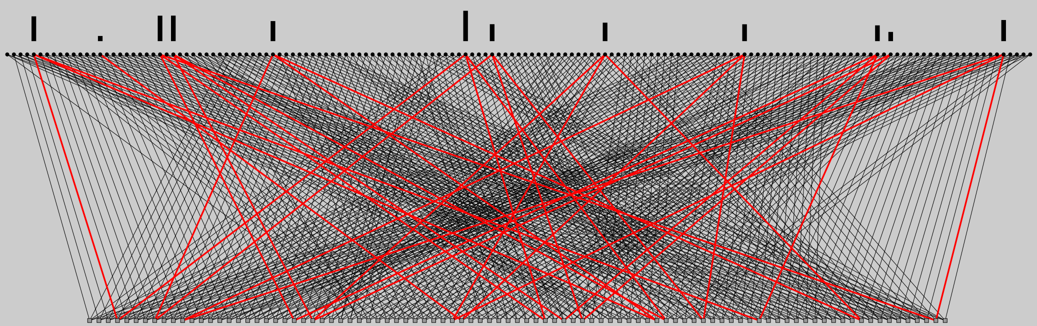
Tanner graph of $[155, 64, 20]$ code

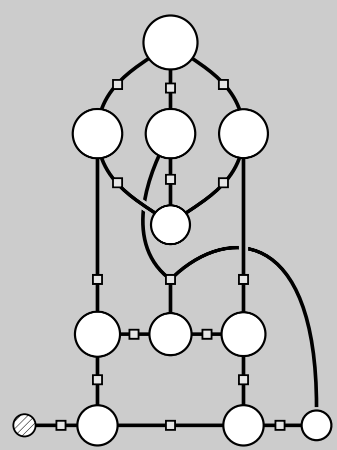


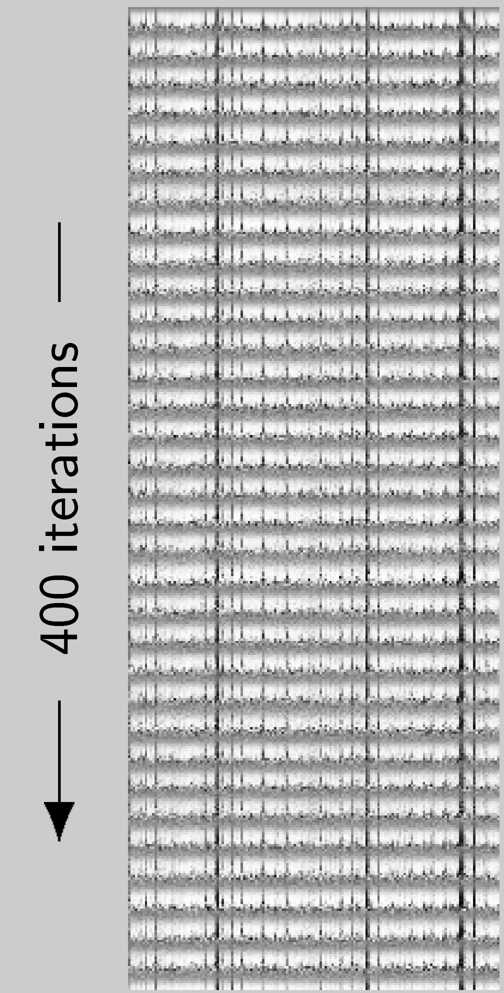
One special subgraph of Tanner graph



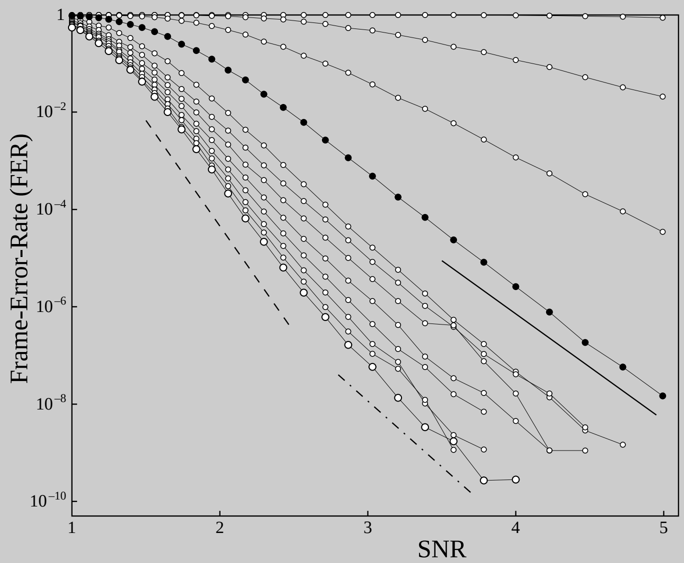
One special subgraph of Tanner graph

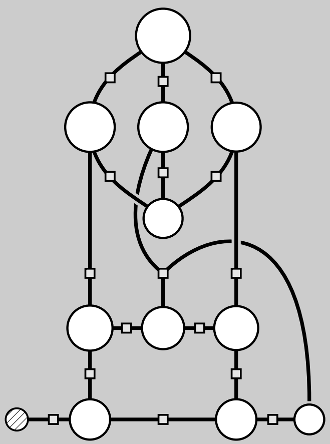




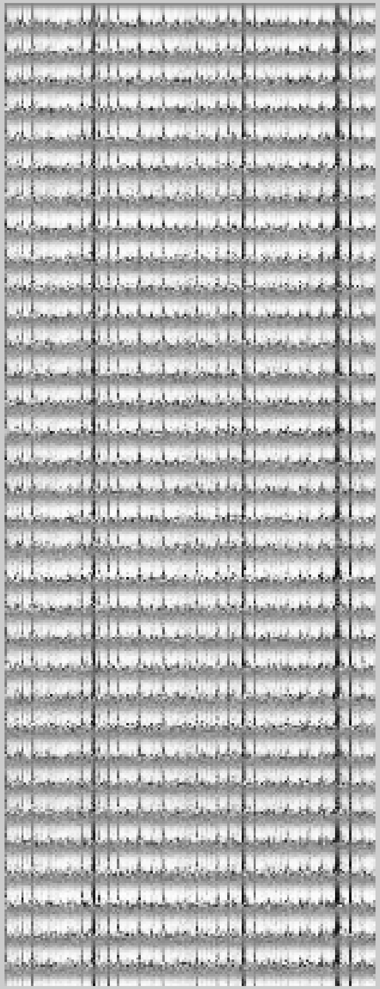


Instanton for Tanner's $[155, 64, 20]$ code





400 iterations



Effective distances:

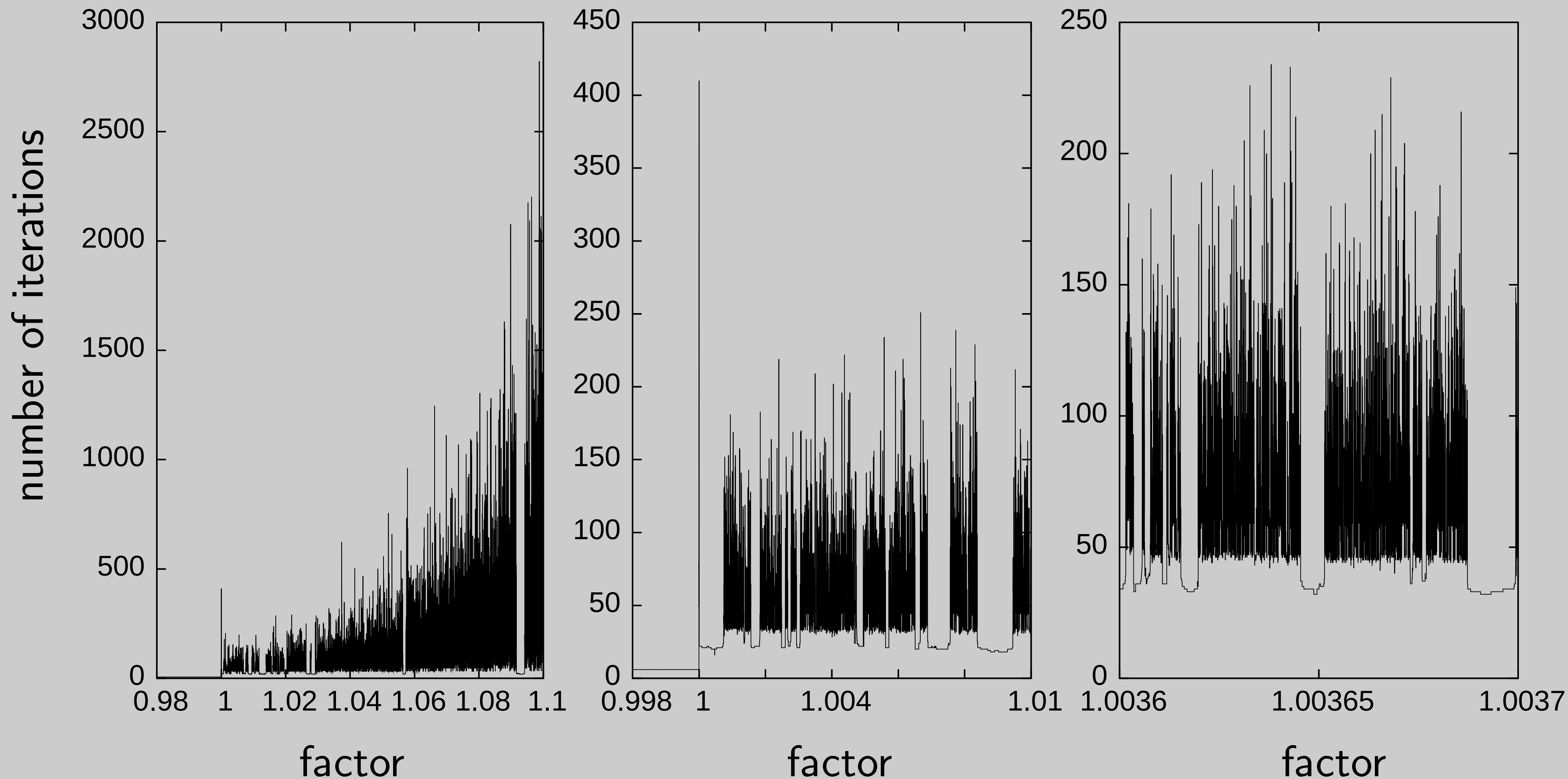
Iterative decoding: 12.5

Linear programming decoding: 16.4

Hamming distance: 20

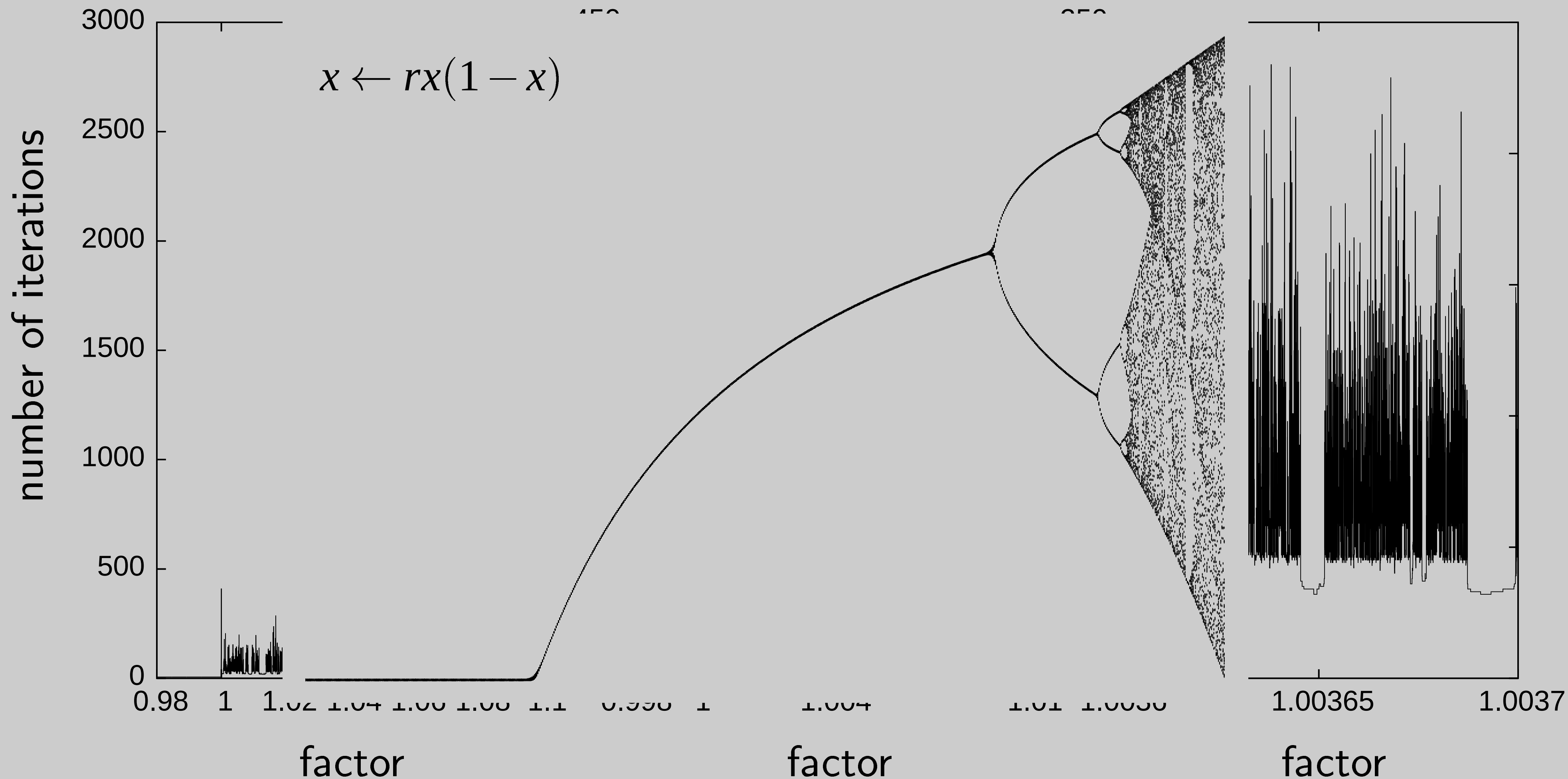
Instanton “robustness”

number of iterations until a successful decoding



Instanton “robustness”

number of iterations until a successful decoding



Smoothed (relaxed, damped) decoding

Iterative scheme (BP): $\eta_{i\alpha}^{(n+1)} = h_i + \sum_{\beta \ni i}^{\beta \neq \alpha} \tanh^{-1}\left(\prod_{j \in \beta} \tanh \eta_{j\beta}^{(n)}\right)$

↓

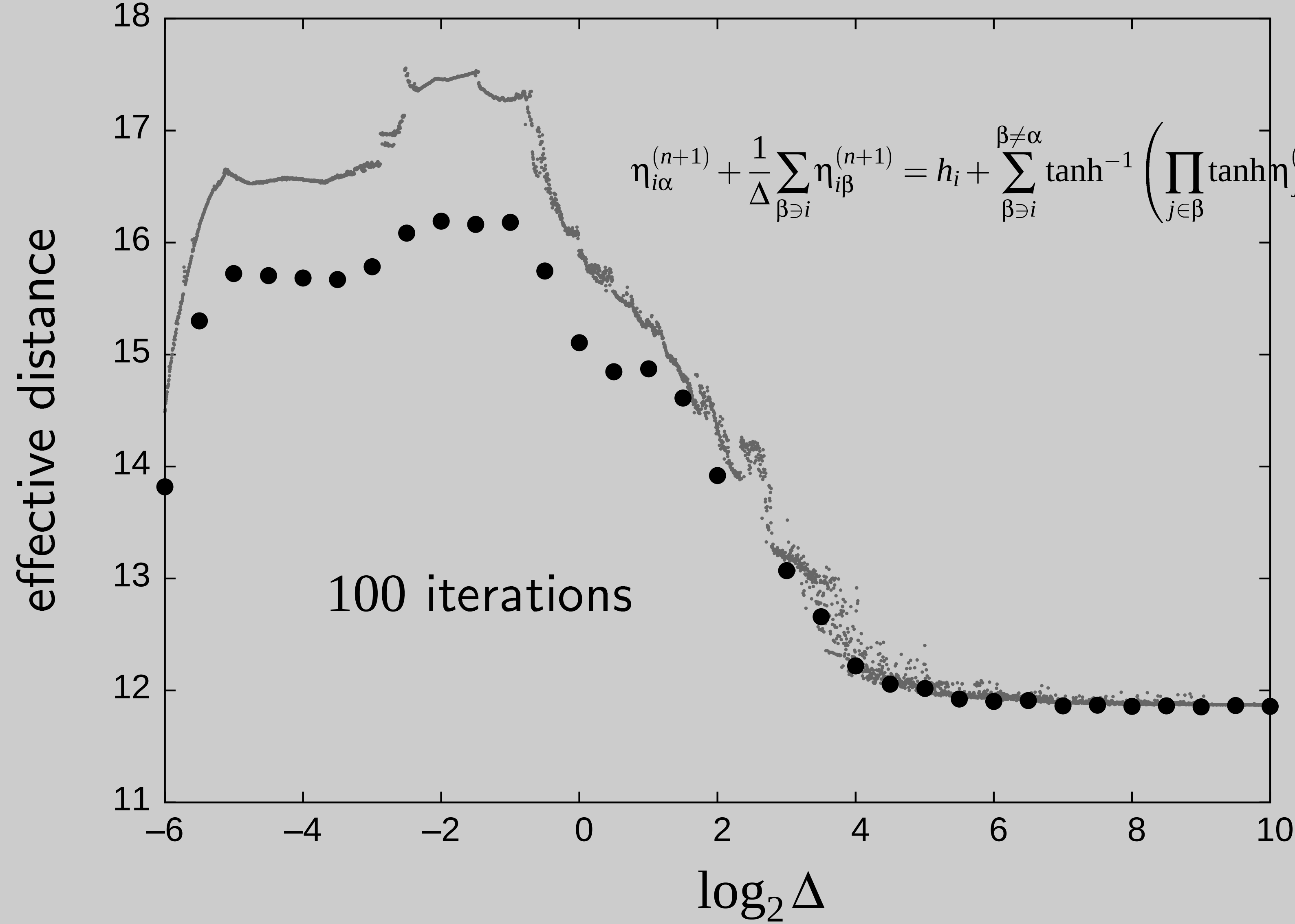
$$\eta_{i\alpha}^{(n+1)} + \frac{1}{\Delta}\sum_{\beta \ni i}\eta_{i\beta}^{(n+1)} = h_i + \sum_{\beta \ni i}^{\beta \neq \alpha} \tanh^{-1}\left(\prod_{j \in \beta} \tanh \eta_{j\beta}^{(n)}\right) + \frac{1}{\Delta}\sum_{\beta \ni i}\eta_{i\beta}^{(n)}$$

$\Delta \to \infty$ — standard BP

$\Delta \to 0$ — slow dynamics

Stepanov, Chertkov, Allerton 2006 [cs.IT/0607112]

Instantons effective distance



$$\eta_{i\alpha}^{(n+1)} + \frac{1}{\Delta}\sum_{\beta \ni i} \eta_{i\beta}^{(n+1)} = h_i + \sum_{\beta \ni i}^{\beta \neq \alpha} \tanh^{-1}\left(\prod_{j \in \beta} \tanh \eta_{j\beta}^{(n)}\right) + \frac{1}{\Delta}\sum_{\beta \ni i} \eta_{i\beta}^{(n)}$$

Summary

- the performance of iterative decoding is determined by most dangerous noise configurations (instantons)

- the fixed point of iterations in decoding is unstable, if the noise configuration is damaging

- the iterative decoding cycles on instantons

- making the iterations smoother helps (shifts the instantons to larger distances)