MATH 583B

due February 21, 2019

Principles and Methods of Applied Mathematics

Section 001, Spring 2019 (instructor: Misha Stepanov)

Homework 8

1. Consider the linear problem $u''(x) + u(x) = f(x)$ with $u(0) = a$ and $u'(\pi/2) = b$ boundary conditions. For which a, b, and $f(x)$ there is a solution?

2. Consider the problem $u(x) - u''(x) = f(x)$ on the semi-line $[0, \infty)$ with $u(0) = a$ and $\lim_{x\to\infty} u(x) = 0$ boundary conditions. For the solution in the form

$$u(x) = aK_0(x) + \int_0^\infty \mathrm{d}y\, K(x|y) f(y)$$

find $K(x|y)$ and $K_0(x)$.

3. Consider the equation $\hat{L}u = f$, where $\hat{L} = \frac{\mathrm{d}}{\mathrm{d}x}$, on the interval $[0, 1]$ with $u(0) = u(1)$ boundary condition (equivalently, you consider $\hat{L}u = f$ on the ring of the length 1, or in the space of 1-periodic functions). (a) What are the conditions on $f(x)$ so that the problem $\hat{L}u = f$ is solvable? (b) Find the "modified" (orthogonalized to the zero modes of $\hat{L}^*$) Green's function $K(x|y)$ and sketch it.

4. Find the Green function $K(x|y) = K(x - y)$ for the equation

$$\left(1 + \frac{a}{N}\frac{\mathrm{d}}{\mathrm{d}x}\right)^N u(x) = f(x)$$

on the whole real line with $u(\pm\infty) = 0$ boundary conditions. The parameter a could be positive, negative, and zero; while N is a positive integer. Find $\int_{-\infty}^{\infty} \mathrm{d}x\, K(x)$. Find $\lim\limits_{N\to\infty} K(x)$.

5. Consider the Poisson's equation $\Delta u = f$ in the strip $-\infty < x < \infty$, $0 < y < \pi$, with the boundary conditions $u(x, 0) = u(x, \pi) = 0$. Find the Green's function.