Homework 9

1. Solve the integral equation

\[ u(x) = 1 + \lambda \int_0^\infty dy \exp(x - 2y) u(y) \]

For which values of \( \lambda \) the iterations would converge?

2. Solve the integral equation

\[ u(x) = x + \lambda \int_0^x dy \ u(y) \]

by (a) differentiating it and solving the resulted ODE; (b) iterations.

3. Solve the integral equation

\[ u(t) = 1 + \lambda \int_0^t d\tau \exp(\tau - t) u(\tau) \]

by (a) differentiating it and solving the resulted ODE; (b) iterations (up to \( \lambda^2 \) term); (c) doing Laplace (or Fourier) transform (the integral in the equation is a convolution).

4. Solve the integral equation

\[ u(x) = 1 + \lambda \int_{-1}^2 dy \ \text{sign}(xy) \ u(y) \]

5. Solve the equation

\[ u(x) = f(x) + \frac{\lambda}{2} \int_0^1 dy \ \text{sign}(x - y) \ u(y) \]

for generic \( f(x) \) and \( \lambda \) (e.g., find the Fredholm resolvent, or reduce it somehow to a [non-homogeneous] differential equation and find its Green function, or whatever you can think of).