$A \subseteq K$

$E : K \longrightarrow K$

$E(0) = 1$

$E(x+y) = E(x)E(y)$

$ecl(A)$ = isolated points of exp. alg. equatn $/\mathbb{Q}(A)$

$f \in K[x_1, \dots, x_n, y_1, \dots, y_n]$

$\tilde{f} = f(x_1, \dots, x_n, Ex_1, \dots, Ex_n)$

$1 \le j \le n$

$\delta_j \tilde{f} = \widetilde{\left(\frac{\partial f}{\partial x_j}\right)} + \widetilde{\left(\frac{\partial f}{\partial y_j}\right)} \cdot Ex_j$

$a \in ecl(A) \iff$

$\exists n \in \mathbb{N} \ \exists f_1, \dots, f_n \in K[x_1, \dots, x_n, y_1, \dots, y_n]$

$\exists a_1, \dots, a_n \in K$

$\mathbb{Q}(A)$

$\tilde{f}_i(\vec{a}) = 0$ fa $1 \leq i \leq n$

$\det\left(\partial_j \tilde{f}_i(\vec{a})\right) \neq 0$

$a = a_1$

CCP: fa A finite

$\mathrm{ecl}(A)$ is countable

<u>Thm</u> (Zilber)

$\forall \lambda > \aleph_0$

$\exists! \ \mathbb{B}_\lambda$ pseudoexponential

field, $|\mathbb{B}_\lambda| = \lambda$

<u>Conj</u> $\mathbb{C}_{exp} \cong \mathbb{B}_{2^{\aleph_0}}$

$$Z(K) := \{ z \in K : \forall x \, [E(x) = 1 \longrightarrow E(zx) = 1] \}$$

$Z(K)$ is a subring of K

SK' :

① $\mathbb{Z}(K)$ has the 1st order theory of $\mathbb{Z}$.

② $\exists\ \varpi \neq 0$

$\ker E = \mathbb{Z}(K) \cdot \varpi$

③ $\forall\ \varpi \in \ker E \setminus \{0\}$

$$\operatorname{trdeg}_{\mathcal{Q}(\mathbb{Z}(K))} \mathcal{Q}(\mathbb{Z}(K))(\varpi) = 1$$

SC : if $\alpha_1, \dots \alpha_n$ are $\mathbb{Q}$-linearly independent, then $\operatorname{tr\,deg}_{\mathbb{Q}} \mathbb{Q}(\vec{\alpha}, \overrightarrow{E(\alpha)}) \geq n$.

$\iff \forall \beta_1, \dots \beta_n$

$$\operatorname{tr\,deg}_{\mathbb{Q}} \mathbb{Q}(\vec{\beta}, \overrightarrow{E(\beta)}) \geq \dim_{\mathbb{Q}} \sum \mathbb{Q}\beta_i$$

If $\beta_i = E(\alpha_i)$

$1 \leq i \leq n$

& $\langle \beta_1, \ldots, \beta_n \rangle$ has

rank m, then

$\dim \sum \mathbb{Q}\alpha_i \geq m$

if $\prod \beta_i^{l_i} = 1$

$\longrightarrow \sum l_i \alpha_i \in \ker E$

SCOK : Schanuel's Conjecture over the kernel

$\forall \alpha_1 \ldots \alpha_n$

$$\operatorname{tr\,deg}_{\mathbb{Q}(\ker E)} \mathbb{Q}(\ker E, \vec{\alpha}, E(\vec{\alpha}))$$

$$\geq \dim_{\mathbb{Q}} \left(\mathbb{Q}\cdot\ker E + \sum \mathbb{Q}\alpha_i \Big/ \mathbb{Q}\cdot\ker E \right)$$

$$= \operatorname{rk}\left(\langle E(\alpha_1), \ldots E(\alpha_n) \rangle \right)$$

Given SK′

<u>If</u> $\ker E = \mathbb{Z}\cdot\varpi$

then $SCOK \Longleftrightarrow SC$

$\Longrightarrow \alpha_1, \ldots, \alpha_n$ $\mathbb{Q}$-linearly independent.

Let ϖ generate $\ker E$

<u>Case 1</u>: $\dim_{\mathbb{Q}}(\mathbb{Q}\varpi + \sum \mathbb{Q}\alpha_i)$

$$= n+1$$

<u>Case 2</u>: $\dim_{\mathbb{Q}}(\mathbb{Q}\varpi + \sum \mathbb{Q}\alpha_i)$

$$= n$$

<u>Case 1</u> :

$$\operatorname{tr\,deg}_{Q} Q(\alpha, E\alpha)$$

$$= \operatorname{tr\,deg}_{Q(\varpi)} Q(\varpi, \alpha, E\alpha)$$

$$\geq n$$

<u>Case 2</u> : $\varpi \in \sum Q\alpha_i$

$$Q(\varpi, \alpha, E\alpha) = Q(\alpha, E\alpha)$$

$$\alpha_n \in \mathbb{Q}\varpi + \sum_{i=1}^{n-1} \mathbb{Q}\alpha_i$$

$$\operatorname{trdeg}_{\mathbb{Q}} \mathbb{Q}(\alpha, E\alpha)$$

$$= \operatorname{trdeg}_{\mathbb{Q}} \mathbb{Q}(\varpi, \alpha, E\alpha)$$

$$= \operatorname{trdeg}_{\mathbb{Q}} \mathbb{Q}(\varpi)$$

$$+ \operatorname{trdeg}_{\mathbb{Q}(\varpi)} \mathbb{Q}(\varpi)(\alpha_1 \ldots \alpha_{n-1}, E\alpha_1 \ldots E(\alpha_{n-1}))$$

$$\geq 1 + n - 1 = n \quad \checkmark$$

$$X \subseteq \mathbb{G}_a^n \times \mathbb{G}_m^n$$

X variety $/_{\mathbb{Q}(\ker E)}$

$\dim X < n$

Goal: say that

if $(a_1, \ldots, a_n, Ea_1, \ldots, Ea_n) \in X(K)$

then there is a multiplicative relation on $E_{a_1}, \ldots, E_{a_n}$.

i.e.

$$\exists (l_1 \ldots l_n) \in \mathbb{Z}^n \setminus \{(0 \ldots 0)\}$$

$$\prod E(a_i)^{l_i} = 1$$

If we could find

$N(X) \in \mathbb{Z}_+$

s.t. ~~o~~ whenever

such a relation there

is also one with

$(l_1', \dots, l_n') \in \mathbb{Z}^n \setminus \{(0, \dots, 0)\}$

$|l_i'| \leq N$

$\prod E(a_i)^{l_i'} = 1.$

$X \subseteq \mathbb{G}_m^g$

$T \leq \mathbb{G}_m^g$ alg. subgroup

we <u>expect</u>

$\dim (X \cap T) =$

$\dim X + \dim T - g$

C a component of $X \cap T$ is called atypical if

$\dim C > \dim X + \dim T - g$

Conjecture on Intersection w/ Tori (CIT)

= Zilber - Pink

for algebraic tori

If $X \subseteq \mathbb{G}_m^q$

is irreducible $/\mathbb{C}$

not contained in

a proper algebraic subgroup, then

$$X^{atypp} = \bigcup_{\substack{C \text{ atypical} \\ \text{component} \\ \text{of } X \cap T \\ \text{for some } T \leq G_m^n}} C$$

is not Zariski closed in X.