1.1 Methods of Factoring

- Standard Form for quadratics is:
$$ax^2 + bx + c$$

- Method of finding signs

  We start with
$$ax^2 \underbrace{+}_{①} bx \underbrace{+}_{②} c$$

  Case 1: If ② is a " − ", then our factors look like (+)(−) or (−)(+).

  Case 2: If ② is a "+", look at ①

  - If ① is a "−", then our factors look like (−)(−).
  - If ① is a "+", then our factors look like (+)(+).

- Helpful Hints (NOTE: we should already know the signs of the factors from the above method)

  1. If there is a common factor in all terms, pull it out first. (i.e. if a, b, c are even, factor out a 2 from all terms first)
  2. If $a = 1$, then our factors will be $(x\pm\quad)(x\pm\quad)$.
  3. If a is prime, then our factors will be $(ax\pm\quad)(x\pm\quad)$.
  4. If $c = 1$, then our factors will be $(\quad\pm 1)(\quad\pm 1)$.
  5. If c is prime, then our factors will be $(\quad\pm c)(\quad\pm 1)$.
  6. If a or c is a square (i.e. 4 or 9 or 16 or 49...), first try its square root in the corresponding place.
  7. $a^2x^2 - b^2$ factors to $(ax + b)(ax - b)$.
  8. $(ax + b)^2 = a^2x^2 + 2abx + b^2$.

- The A-C method

  Multiply $a \cdot c$, then factor the product <u>completely</u> and see what combinations of factors sum to be b.

1.2 Examples

1. Solve $2x^2 - 8x + 6 = 0$.

   Solution 1: We solve by factoring. Using hint 1, we notice that 2 is a common factor, so we can pull it out. Hence, we have $2(x^2 - 4x + 3)$. To determine the signs, we look at the equation $2(x^2 \underbrace{-}_{①} 4x \underbrace{+}_{②} 3)$.

   Since ② is a "+", we look at ①. Since ① is a "−", we know our factorization will look like $2(\quad - \quad)(\quad - \quad)$. Notice that in this slightly factored form, $a = 1$ and $c = 3$. Using hints 2 and 5, we know that we can write $2(x-1)(x-3)$. So, $2(x-1)(x-3) = 0$. Thus, $x = 1$ or $x = 3$.

   Solution 2: Using the A-C method, we consider $2 \cdot 6 = 12$. Now we look at the factors of 12 and decide which ones add up to be 8. The factors of 12 are

1&12	$1 + 12 = 13$
2&6	$2 + 6 = 8$
3&4	$3 + 4 = 7$

   Notice that $6 + 2 = 8$ and $-6 + (-2) = -8$. We choose the combination with sum of -8 since -8 is our middle term. So we know that after we foil, we should get $2x^2 - 2x - 6x + 6$. **Factoring by grouping**, we see that

   $$\begin{aligned} &2x^2 - 2x - 6x + 6 \\ &2(x^2 - x - 3x + 3) && \text{Factor out the constant 2} \\ &2[(x^2 - x) + (-3x + 3)] \\ &2[x(x-1) + 3(-x+1)] && \text{Factor out GCF of each term} \\ &2[x(x-1) - 3(x-1)] && \text{Factor out } (-1) \text{ from the right hand term} \end{aligned}$$

   Notice that the Greatest Common Factor of these two terms is $(x-1)$. Since $(x-1)$ is the GCF, we can factor it out. Then what is left are the coefficients of the $(x-1)$ terms, which when combined are $(x-3)$. Thus we have

   $$2[(x-1)(x-3)]$$
   $$2(x-1)(x-3)$$

   So, $2(x-1)(x-3) = 0$. Hence, $x = 1$ or $x = 3$.

2. Factor $25x^2 + 80x + 64$. *FOIL* to check.

   Solution: From our helpful hint 6, we see that $25 = 5^2$ and $64 = 8^2$. So we should try 5 and 8 first. So, $(5x+8)(5x+8)$. *FOIL*ing to check, we see that this is the factorization.

3. Solve $10x^2 - 26x - 12 = 0$.

Solution: Hint 1 reminds us to factor out the GCF first. Hence, we get that

$$2(5x^2 - 13x - 6) = 0.$$

Now we can use our method of determining signs. Since the second sign is a "$-$", we know our factors will look like $2(\quad + \quad)(\quad - \quad)$. Notice that 5 is prime, so using helpful hint 3, we know our factors will look like

$$2(5x+\quad)(x-\quad) \qquad \text{OR} \qquad 2(x+\quad)(5x-\quad)$$

Say we choose to start with the left option and see if we can factor it. Next we consider the factors of 6. The factors are $6 \cdot 1$ and $2 \cdot 3$. Since our b term is negative, the larger of the two factors of six should go with the negative sign. If we try 6 and 1, we FOIL to get

$$2(5x+1)(x-6) = 2(5x^2 - 29x - 6) \neq 2(5x^2 - 13x - 6)$$

Since that didn't work, we try the other factors of 6. So, since $3 > 2$, we put the 3 with the negative sign, and we get that

$$2(5x+2)(x-3) = 2(5x^2 - 13x - 6)$$

as desired. So then

$$\begin{aligned} 10x^2 - 26x - 12 &= 0 \\ 2(5x^2 - 13x - 6) &= 0 \\ 2(5x+2)(x-3) &= 0 \end{aligned}$$

That means either $5x + 2 = 0$ or $x - 3 = 0$. Solving for x is both cases, we see that $x = -\frac{2}{5}$ or $x = 3$.

Note: If we had chosen the right hand option (using $2(x+\quad)(5x-\quad)$), we would have gone through all the possibilities of moving the factors around and found that none worked. Then we would have moved on to the left option. In some cases you can guess which is probably best, but usually its hard to determine. The general advice is if you can't see right away that you should use one option over another, then just choose one and work with it until you successfully factor, or exhaust all possibilities of the factors.

4. Factor $12x^2 - 63x + 12 + 3x^2$.

Solution: First we need to put the quadratic in standard form. Combining like terms, we get $15x^2 - 63x + 12$. Now that the equation is in standard form, we can pull out the GCF. So we have

$$3(5x^2 - 21x + 4)$$

Using our method for determining signs, we see that the second sign is a "+", so we look at the first sign and put it in both factors. Since ① is a "−", we know our factors should look like (−)(−). Since $a = 5$ is prime, we can write

$$3(5x- \quad)(x- \quad)$$

The factors of 4 are $4 \cdot 1$ and $2 \cdot 2$. In order to get a large negative number for our b term, we should choose a large factor to multiply by the $5x$ in order to get close to our b term. Since 4 is the largest term available, we try that first. Thus, we have $3(5x - 1)(x - 4)$. FOILing to check, we see that

$$3(5x - 1)(x - 4) = 3(5x^2 - 21x + 4)$$

as desired.
Thus, our factorization is $3(5x - 1)(x - 4)$.