1.1 Rules of Roots

The following rules are true for all x, y except where specified.

- $\sqrt{x^2} = |x|$
- $\sqrt{-1} = i$
- $\sqrt{-x} = i \cdot \sqrt{x}$
- $\sqrt{x} \cdot \sqrt{y} = \sqrt{x \cdot y}$
- $\dfrac{\sqrt{x}}{\sqrt{y}} = \sqrt{\dfrac{x}{y}}$ when $y \neq 0$
- $\sqrt{x^3} = x \cdot \sqrt{x}$
- $\sqrt[3]{x^3} = x$
- $\sqrt[3]{x^4} = x \cdot \sqrt[3]{x}$

  In general:

  - $\sqrt{x} = x^{\frac{1}{2}}$
  - $\sqrt[n]{x} = x^{\frac{1}{n}}$
  - $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

  Things to be careful of:

- $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$
- $\sqrt{x-y} \neq \sqrt{x} - \sqrt{y}$

1.2 Quadratic Formula

- Standard Form for quadratics is:

$$ax^2 + bx + c$$

- Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

  A general rule for plugging in the a, b, c in the quadratic formula is to put parenthesis around each value when you plug it in. This will help to keep the signs straight.

1.3 Examples

1. Solve $2x^2 - 8x + 6 = 0$ using the quadratic formula.

   Solution 1: First, we notice and write that $a = 2$, $b = -8$, and $c = 6$. So,

   $$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(6)}}{2(2)} = \frac{8 \pm \sqrt{64-48}}{4} = \frac{8 \pm \sqrt{16}}{4} = \frac{8}{4} \pm \frac{\sqrt{16}}{4} = 2 \pm \frac{4}{4} = 2 \pm 1$$

   So, $x = 2 + 1 = 3$ or $x = 2 - 1 = 1$.

2. Find the zeros of the function $5x^2 + x - 2$.

   Solution: We want to find where $5x^2 + x - 2 = 0$. Notice that this is not easily factorable, so we have to use the Quadratic Formula. So, using $a = 5$, $b = 1$, $c = -2$,

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(1) \pm \sqrt{(1)^2 - 4(5)(-2)}}{2(5)} = \frac{-1 \pm \sqrt{1+40}}{10} = \frac{-1 \pm \sqrt{41}}{10} = -\frac{1}{10} \pm \frac{\sqrt{41}}{10}$$

   These aren't very nice numbers, but we can't do anything about that. So, $x = -\frac{1}{10} + \frac{\sqrt{41}}{10}$ or $x = -\frac{1}{10} - \frac{\sqrt{41}}{10}$.

3. Solve $x^2 + 2x + 2 = 0$.

   Solution: Since this is not easily factorable, we need to use the quadratic formula. First, we identify $a = 1$, $b = 2$, and $c = 2$. So,

   $$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(2)}}{2(1)} = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = -\frac{2}{2} \pm \frac{\sqrt{-4}}{2} = 1 \pm \frac{2i}{2} = 1 \pm i$$

   So $x = 1 \pm i$. Thus, we have complex roots. Hence, $x = 1 + i$ or $x = 1 - i$.

4. Find the roots of the following polynomial using the Quadratic Formula

   $$P(x) = -2x^2 + 4x - 2$$

   Solution: Notice that $a = -2$, $b = 4$, and $c = -2$. So,

   $$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(-2)(-2)}}{2(-2)} = \frac{-4 \pm \sqrt{16-16}}{-4} = \frac{-4}{-4} \pm \frac{0}{-4} = 1 \pm 0 = 1$$

   So $x = 1$ is the only root of this polynomial.

5. Solve $x^2 + 8x + 17 = 0$.

   Solution: Notice that $a = 1$, $b = 8$, $c = 17$. So,

   $$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(17)}}{2(1)} = \frac{-8 \pm \sqrt{64-68}}{2} = \frac{-8}{2} \pm \frac{\sqrt{-4}}{2} = -4 \pm \frac{2i}{2} = -4 \pm i$$

   So $x = -4 + i$ or $x = -4 - i$.