CORRECTION TO “$p$-DESCENT IN CHARACTERISTIC $p$”

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Felipe Voloch points out that the description of Sel$(K, V)$ in part (a) of Proposition 3.3 is not correct when $A$ is a $(p - 1)^{st}$ power. Under the hypotheses of the Proposition, Lemma 1.2 shows that Sel$(K, V) \cong H^1(X, \text{Ker } V)$. There is an exact sequence

$$0 \to \text{Ker } V \to \mathcal{O}_X \xrightarrow{\varphi_A} \mathcal{O}_X \to 0$$

where $\varphi_A(x) = x^p - Ax$. Taking cohomology yields an exact sequence

$$0 \to k/\varphi_A(k) \to \text{Sel}(K, V) \to \{\eta \in H^1(X, \mathcal{O}_X) | Fr(\eta) = A\eta\} \to 0.$$

When $A$ is not the $(p - 1)^{st}$ power of an element of $k$, then $k/\varphi_A(x)$ is zero and the statement is correct. However, when $A$ is a $(p - 1)^{st}$ power, $k/\varphi_A(k)$ has order $p$ and the Selmer group is larger than stated.

Our thanks to Voloch for pointing out this mistake.

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