# 1 Abstract

Plato considered the straight line and circle the only “perfect” geometrical figures, and this restricted the tools available for the constructions of ancient Greek geometry to two: the unmarked ruler and the compass. The Greeks could do many things with these tools, ranging from dividing a line into arbitrarily many equal parts to constructing a regular 17-gon. However, they never formulated their ideas algebraically and thus many unanswered questions remained. Can one trisect an arbitrary angle, can one square the circle, and for which values of n is the regular n-gon constructible are a few of the more well known ones.

The third of these questions is the focus of our research. Gauss definitively proved for a regular n-gon to be constructible, n must be a product of powers of 2 and Fermat primes. Since then considerable effort has been spent in the search for algorithms for these constructions. Algorithms exist for the values of n corresponding to the first four Fermat primes, but the fifth prime, $p = 65537$, does not possess a definitive algorithm for its construction. This paper uses the techniques of modern abstract algebra, linear algebra, and Galois theory to provide a simple algorithm for the construction of any n-gon. The properties of a special $\mathbb{Q}$-automorphism are exploited to obtain a tower of quadratic extensions of the rationals, which can be directly solved for a radical expression for $\cos \frac{2\pi}{n}$, from which the corresponding n-gon may be constructed. The tower itself will also be manipulated to give another construction for a regular n-gon.