Understanding Otoacoustic Emissions Generation

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1 Introduction

The primary function of the ear is to convert acoustic stimuli to neural signals. The pressure from sound waves reaches the ear and is transformed into mechanical energy. This is done by forcing the vibration of stereociliary bundles in the inner ear, or cochlea. This mechanical energy is then transformed into electrical energy and neural impulses to be sent to the brain [1]. Surprisingly, the cochlea is also capable of producing and emitting sound. These sounds are known as otoacoustic emissions (OAEs). They can be spontaneous or evoked by an external stimulus. The generation mechanisms of these OAEs is unclear, but one hypothesis is that they originate from the outer hair cells located in the cochlea [2].

The distortion product emissions (DPOAEs) are one form of evoked OAEs. For this type of emission, the inner ear is stimulated by two tones at frequencies $f_1$ and $f_2$. A spectral analysis can be performed on the resultant sound wave revealing the DPOAE. The resultant distortion products are predictable. They are dependent on one or both of the input signals. The goal of this research is to investigate whether the behavior of the otoacoustic emissions for the non-monotonic growth functions can be explained as the result of a single source mechanism or if more mechanisms are necessary.
2 Fourier Analysis

Otoacoustic emissions contain useful information for analysis that is not readily evident from a graph of their wave forms. One such hidden piece of data is the frequencies contained in the wave. Fourier analysis provides a global approximation for this wave form using periodic functions. The equations derived from a Fourier approximation can easily be analyzed with a spectral analysis to determine the useful information. The general form of the OAEs for a Fourier expansion with period b is

\[ f(x) = a_0 + \sum_{k=0}^{n} a_k \cos\left(\frac{2\pi k x}{b}\right) + \sum_{k=0}^{n} b_k \sin\left(\frac{2\pi k x}{b}\right) \] (1)

This approximation can be used for functions with any period. An important aspect of this approximation is that it can be used to find the different harmonics in the wave. This means that the energy of the various frequencies in the wave can be calculated. The amplitude of the \(k^{th}\) harmonic is found to be

\[ A_k = \sqrt{a_k^2 + b_k^2} \] (2)

The energy of the constant term \(a_0\) from the Fourier approximation is

\[ A_0 = \sqrt{2} a_0 \] (3)

The ability to analyze the frequencies in an OAE is useful in the understanding of how the input sound wave is transformed in the cochlea. The Fourier analysis provides useful information about the DPOAEs. Software such as Matlab allows for quick analysis of sounds with the fast Fourier transform. The fast Fourier transform quickly and efficiently performs a spectral analysis on a sound wave to determine the frequencies of the OAE and their amplitude with the software. The fast Fourier transform is useful because it dramatically reduces the number of calculations necessary to reach an approximation of a function.

3 Input-Output Functions

Input-output functions are another method used to analyze OAEs and DPOAEs. Input-output graphs display the relationship between the input to a system
and the resulting output. Stimulating sounds are the input to the cochlea which is nonlinear. The OAEs are the output. DPOAEs have two sound stimuli for the input. The input for a DPOAE as a function of time, $I(t)$, can be described as

$$I(t) = A_1 \cos(2\pi f_1 t) + A_2 \cos(2\pi f_2 t)$$ (4)

Analysis of the input and output functions of the cochlea is necessary to understanding the how the system manipulates the functions. The goal of the research is to determine whether one mechanism, specifically the outer hair cells in the ear, is sufficient to explain the nonlinear phenomenon of the cochlea, or inner ear. Otherwise there would be several competing systems transforming the input signals. A single system would be easier to investigate using basic mathematical methods. Examining the behavior of the hair cells in response to the stimulus and the resultant electric potential (mV) demonstrates how the output of the system affects the ear [3]. A function for the input signals can be used with a level dependent input-output function for the cochlea to find an output function. This output function can be compared with the hair cell data to determine if the hair cells have a significant role in the generation of OAEs. The nonlinear least squares method will be used to fit a function to the output data. This will be necessary in order to analyze the output function fully [4].

4 The Least Squares Method

The least squares method is a mathematical procedure for determining the best fitting function for a set of data. The data that will be examined is mouse OAE data taken in a lab by a collaborator on the project at the Massachusetts Institute of Technology. Several different mice were used with varying intensity levels of sound in the course of the experiment. A nonlinear least squares method will fit a function to this data. It does so by creating an error function. The error function is equal to the square of the vertical deviation of a possible fit for a given set of parameters and the data. The deviation is squared in order to eliminate problems due to signs. Symbolically, this error function, $R^2(a,b)$, for a linear least square for n data points can be described as
The data that will be used as the basis for the approximate function does not have a linear relationship however. It appears to be sigmoid, or possibly a hyperbolic tangential function. It will require several more variables. Symbolically, this nonlinear error function, $R^2(a,b,c,d)$, for $n$ data points can be described as

$$R^2(a, b) = \sum_{i=1}^{n} (y_i - (a + bx_i))^2$$

The global minimum for the error function reveals where this deviation is the smallest and thus the best fit for the data. This minimum can be found for a linear graph by taking the partial derivative with respect to $a$ and $b$. Partial derivatives with respect to each variable will be taken. The partial derivatives will be used to determine the minimum error and the appropriate approximation [5]. Since this equation is more complex than the linear equation, Matlab will be used to find the minimum error. A program must be created that can examine the partial derivatives in a given range. A brute force method will be used to measure all the partial derivatives in the range. The area with the least error will be examined further until the smallest error is found.

5 Conclusion

Once the approximate equation is found, it can be used with the input equation to determine if a single source is sufficient for explaining the non-monotonic growth functions. The input will be entered into the input-output function and the result compared with the output function. A simple graph should be enough to come to a conclusion regarding the original question for the research. The output function will be further analyzed to see if the current conjectures regarding their shape holds true at different intensities of sound. Specifically, we will investigate whether all frequencies of the DPOAE contain notches in the graph comparing the hair cell displacement and their receptor potential. Currently, only certain frequencies are believed to contain these notches. The equations for these graphs however appear to demonstrate a possibility for notches in all frequencies. This hypothesis
can be investigated analytically with the equations and numerically with the mouse DPOAE data.

References


