Increasing the Number of Mathematics Majors

By William Yslas Vélez

The last couple of decades have seen a decline in the number of undergraduates pursuing degrees in mathematics. During the same time, quantitative methods have been increasingly present in the sciences, necessitating a more rigorous mathematical training for students interested in pursuing a research career. It is perfectly reasonable to suggest that future researchers in the sciences should be majoring both in mathematics and in the sciences. Moreover, an undergraduate mathematics major, coupled with computing skills and a knowledge of basic science, is an employable combination. When thinking of students majoring in mathematics, the fact that such students can be employed with an undergraduate degree must be uppermost in our minds. We must not think of the undergraduate degree in mathematics as the first step towards the doctorate.

If we are to increase the number of students in this country that are more mathematically literate, then we must address the question of how we are to entice these students to include more mathematics in their undergraduate curriculum. One activity that readily comes to mind is *outreach*. By this I mean activities by mathematics departments to reach out to pre-college students. The purpose of these activities is to acquaint these students with the importance of mathematics and to introduce them to the possibility of choosing mathematics as their major when they arrive in college. These activities are certainly important. I would like to see a more concerted outreach activity, specifically aimed at those students taking calculus in high school. Some of these students have mathematical talent and an interest in mathematical thinking, but if no one informs them of the possibility of majoring in mathematics, they will choose other majors. Students who begin their university studies by enrolling in second or third semester calculus have the added benefit that they can complete the requirements for the mathematics major in less than four years. This gives these students the opportunity to either broaden their undergraduate education by selecting another major if they so choose, or to take graduate courses in mathematics as undergraduates, thereby making them more competitive when they apply for graduate programs in the mathematical sciences.

My own department began an outreach project that we call *The High School Calculus Visitation Project*. Our goal is to visit every high school calculus class in the city of Tucson at least once each year. The visitation team includes a faculty member, a graduate student and an undergraduate. The visit consists of presenting an interesting application of calculus. The team takes overheads showing the schedule of mathematics classes that a mathematics major might take in college. This is important because it shows the students that a mathematics major can be completed by only taking one or two mathematics classes each semester. The undergraduates chosen for these visits have had summer internships and they talk about their summer travels and activities.

I believe that high school calculus classes represent an important reservoir of students that could be attracted to the study of mathematics. Students who take calculus in high school have the option of not taking any mathematics when they arrive in college. These advanced students are given the opportunity to opt out of college level mathematics. If the mathematics community does not reach out to these high school calculus classes to inform them of the benefits of a mathematical education, then we will continue to ignore this important population. I would like to see mathematicians develop materials and programs specifically aimed at these students and their teachers.

David Bressoud has been looking into the number of high school students taking calculus and evaluating the impact that this could have on our college calculus courses. He conservatively estimates [private communication] that there are currently over 500,000 high school students enrolled in calculus, approximately equal to the number of college students enrolled in first semester calculus. Of those who took the AP Calculus BC exam, just under 12,500 were in 8th, 9th, 10th, or 11th grade. Of those who took the AB exam, just under 34,000 were in 8th, 9th, 10th, or 11th grade. These students, when they arrive in college, could easily complete the requirements for the mathematics major in three years, giving them time to either have a double major or to take graduate mathematics courses.

Compare these high school numbers with the output of our undergraduate programs. Approximately 11,000 students graduate as mathematics majors each year. The amount of mathematical talent in high schools that these numbers represent is enormous. If we could tap into the talent of these high school students, we could easily double the number of mathematics majors graduating from our colleges. This pool of students is simply too large to ignore and I again ask the mathematical community to develop outreach efforts to these high school calculus classes.

There are many different kinds of outreach activities that mathematics departments can sponsor. Certainly departments should have a varied portfolio of such activities. This is particularly important for mathematics departments that have graduate programs. Having graduate students involved in outreach should be part of the training of our future professors and researchers.

Though outreach is an important activity, it is time-consuming and requires faculty to travel outside of the university. Many faculty members are resistant to participating in such activities. There is an even more important activity that can serve to increase the number of mathematics majors, and that is *inreach*. We should be reaching into our own mathematics classes and encouraging the students that are sitting in front of us to pursue mathematics as a major. Mathematics holds a unique place

in undergraduate studies. Students entering our universities as engineering or science majors must take courses in mathematics. We have a ready made pool of students taking our courses. Why can't we encourage those students to add mathematics as another major? If every calculus or differential equations class resulted in just a few more mathematics majors, we would quickly double the number of mathematics majors.

In the late 1980s, I began my own inreach efforts. I had a very modest goal: I wanted to help minority students succeed in their first semester calculus course. This course is critical for students pursuing engineering and science degrees. As I worked with minority students, my own view of this interaction began to change. Yes, I still wanted them to succeed, but I also wanted these students to take more mathematics, and to consider majoring in mathematics. This initial work evolved into my *Calculus Advising Program for Minority Students.*

Each semester I obtain a listing of all minority students enrolled in our three-semester calculus sequence. Students from this list are called and asked to come in to see me for a twenty-minute appointment. These advising sessions begin about two weeks before the semester starts. Now that email is available, I send these students a one-page letter with a list of calculus resources. The letter lists the websites for the course syllabi, the homework assignments and the lists of final exam questions for the calculus courses. The final exam questions I find particularly useful. Many students take calculus in high school and I am often asked by the student whether or not they should skip the first or second semester calculus course. My suggestion is to work on the final exam questions for these courses (which are on-line). If they can do more than 70% of these problems then this is a good indication that they could skip the course.

In the twenty minutes that I have with the students, I inquire as to their career plans, go over their schedule of courses, suggest changes, and introduce them to university resources. I encourage them to create a résumé. For this purpose I have created a sample résumé that I send them by email as a template for them to create their own résumé. I also show the students our website that contains summer internship information. I want students to begin thinking about applying for summer internships as soon as possible. I also mention the fact that if the student is interested in pursuing an advanced degree, then as long as their grades are good, funding is available, and I give examples of the type of funding. It is surprising how many students are unaware of the fact that summer internships pay them, and not the other way around.

I use these advising sessions as a recruitment tool for the mathematics major. If a student walks into my office enrolled in calculus and has not declared a major, then with high probability that student leaves my office as a mathematics major. Over the last few years about 15% of our mathematics majors have been minority students. In 2005, five Native Americans received undergraduate degrees in mathematics.

In August 2003 I accepted the position of Associate Head for Undergraduate Affairs in our department. I am now in charge of our entire undergraduate mathematics major program. I am applying the lessons that I learned in working with minority students to all of our students.

The work that I have done with minority students has taught me important lessons.

- Provide timely information to students. Help them to understand the system and future opportunities. Even good students need attention and advice.
- The transition from high school to university is brutal. Examine ways to ease this transition.
- Students oftentimes choose engineering because they liked mathematics in high school. These students should also be mathematics majors.
- There are amazing students arriving at our universities and we are not paying attention to them.
- If students are majoring in X, then by adding mathematics as another major, X becomes more flavorful.
- We, as mathematicians, need to take the initiative to communicate the necessity of studying mathematics.

In my new position, I have access to student records. I look over enrollments in our classes and I pay particular attention to first year students. When I see a first year student enrolled in third semester calculus or our sophomore level course in linear algebra, I send that student a message. "I see that you are majoring in X. That is wonderful. Since you are advanced in your mathematics, you could easily add mathematics as another major and still graduate in four years. Stop by my office to talk about this so that I can explain the benefits of being a mathematics major." I have created various templates for these purposes that allow me send out messages without having to create individual messages.

When grades are posted I do the same thing. I look over grades and look for students who have done well. I send them messages suggesting that they add mathematics as another major. I also suggest that they enroll in some particular mathematics course and I invite them to stop by my office to talk.

I also look at enrollments in all of our senior level courses. All of our courses are taught in sections of fewer that 35 students. I look at individual students and I go over their grades and course selection since they arrived at our university. For those non-mathematics majors who are doing well, I send them a message praising them for their progress, describing the remaining courses that they need in order to complete the mathematics major. Of course, there are also students who are not doing well. I often send them a message, suggesting a different sequence of mathematics courses that they might take and inviting them come in and see me.

One of the tools that I use to entice students to major in mathematics is the structure of the mathematics major that I inherited. There are seven options in the mathematics major. Some

of these options are designed for help students go on to graduate school in the mathematical sciences, while others prepare students for the workforce or for graduate school in other sciences. These options are very similar, but having them announces to the students the wide applicability of mathematics.

Another important tool that I use is summer internships. For the last two summers over 30 of our undergraduate students have held some form of paid summer internship. Helping students find internships is critical. Maintaining contact with previous graduates who are now in the workforce is important in finding summer positions for undergraduates. Having an updated resume is a necessary ingredient for this.

A new important source of support for mathematics majors comes from the biological sciences. There are growing opportunities for mathematics majors in laboratories on your campuses and I would recommend establishing contact with faculty in the biological sciences to find positions for mathematics majors.

I had a modest goal of doubling the number of mathematics majors when I took on this new position. Our department has had about 300 mathematics majors each year for the last ten years. As of December 2005, we had over 450 mathematics majors. In December 2005 we looked over our undergraduate majors. Over 100 of these students had GPAs over 3.5. Of these, 57 had perfect GPAs of 4.0! These are good students. It is also impressive how many of these students have double majors. All of our senior level courses are now running at capacity. In fact, in fall 2005, we had to run two sections of abstract algebra and real analysis.

Students want to study mathematics, but they need to know that the mathematics major is a viable option. Mathematics departments need to take on the responsibility of presenting this information to their students.

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